- **Couples**

A special case of moments is a couple. A **couple** consists of two parallel forces that are equal in magnitude, opposite in sense and do not share a line of action. It does not produce any translation, only rotation. The resultant force of a couple is zero. BUT, the resultant of a couple is not zero; it is a pure moment.

Consider the action of two equal and opposite forces **F** and $-\mathbf{F}$ a distance d apart, as shown in the Figure. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple **M**. This couple has a magnitude

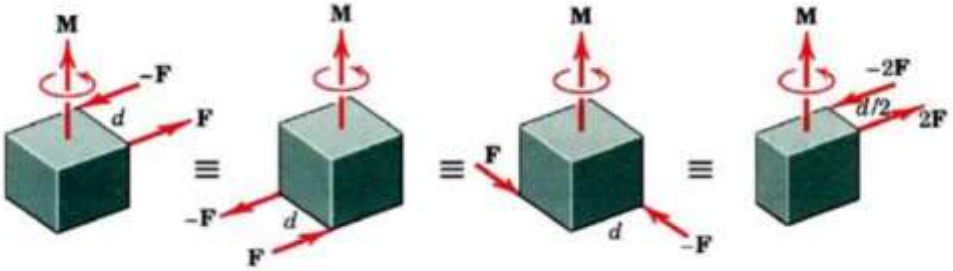
$$M = F(a + d) - Fa$$

or

$$M = Fd$$

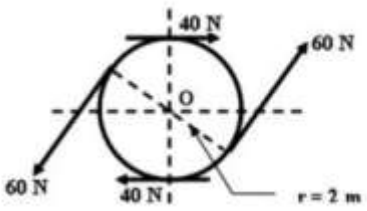
Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance a which locates the forces with respect to the moment center O. It follows that the moment of a couple has the same value for all moment centers.

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected in the force act in a different but parallel plane. The figure below shows four different configurations of the same couple M. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.



Ex (1)

Compute the magnitude and direction of the resultant couples action on the body shown



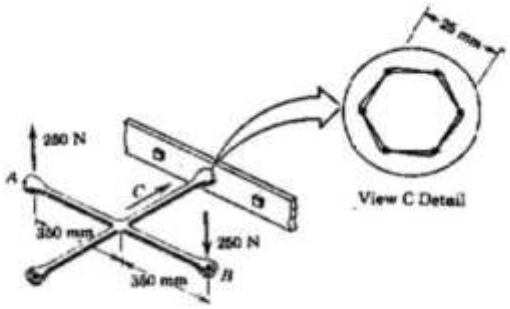
Solution :

Mc = 60 * 4 – 40 * 4

= 240 – 160 = 80 N .m

Ex (2)

A lug wrench is used to tighten a hex-head bolt , Determine the magnitude (F) of the equal forces exerted on the six contact points as shown in fig.



Solution

On the lug wrench :

Mc = F * d

= 250 * 350 *2

= 175000 N . mm

On the nut :

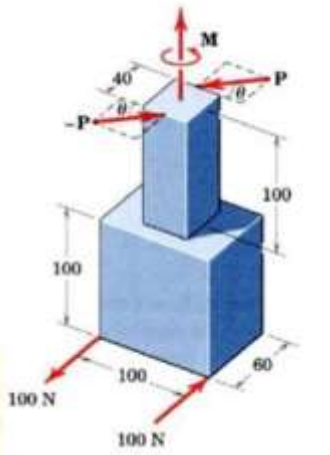
Mc = 3 F * d

175000 = 3 F * 25

F = 175000 / 75 = 2333.33 N

Ex (3):

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces P and –P, each of which has a magnitude of 400 N. Determine the proper angle θ.



Dimensions in millimeters

Solution:

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is:

$[M = Fd]$ $M = 100(0.1) = 10\ \text{N}\cdot\text{m}$

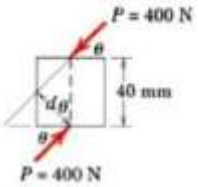
The forces **P** and **−P** produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

Equating the two expressions gives

$$10 = 400(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ$$



Ex (4):

Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at *O* and a couple.

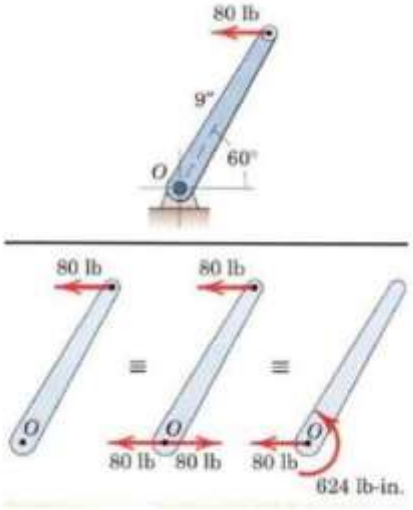
Solution:

We apply two equal and opposite 80-lb forces at *O* and identify the counterclockwise couple

[M=F.d]

M=80 (90 sin 60°)= 624 lb-in.

Thus, the original force is equivalent to the 80-lb force at O and the *624 lb-in* couple as shown in the third of the three equivalent figures.



Ex (5):

In the design of lifting hook , the action of the applied force (F) at the critical section of the hook is a direct pull at (B) and a couple . if the magnitude of the couple is (4000 lb.ft) , Determine the magnitude of the force (F) .

Solution

Mc = F * d

F = Mc / d

= 4000* 12 / 4

= 12000 lb