

## 2-Weight-Volume Relationships (Phase Relationships)

Partially saturated soil (*three-phase soil*) is composed of solid (soil particles), liquid (usually water), and gases (usually air). The spaces between the solid are called voids. the soil water is commonly called pore water and it plays a very important role in the behavior of soils under load. If all voids are filled with water, the soil is saturated (*two phases*). Otherwise, The soil is unsaturated. If all the voids are filled with air. The soil is said to be dry (*two phases*).

### 2-1 Weight-Volume Relationships

Figure(1) shows an element of the soil of volume  $V$  and weight  $W$  as it would exist in a natural state. To develop the weight -Volume relationships. We must separate three phases (that is, solid, water, and air). Thus the total volume of a given soil sample can be expressed as :

$$V = V_s + V_v = V_s + V_w + V_a \dots\dots\dots(1)$$

where  $V_s$ : vol. of soil,  $V_v$ : void,  $V_a$ : air

The total weight of the sample is as:

$$W = W_s + W_w \dots\dots\dots (2)$$

where  $W_s$ : weight of soil,  $W_w$ : weight of water.

The volume relationships commonly used for the three phases in a soil element are void ratio, porosity, and degree of saturation. void ratio ( $e$ ) is defined as the ratio of the volume of the void to the volume of solids. Thus,

$$e = \frac{V_v}{V_s} \dots\dots\dots(3)$$

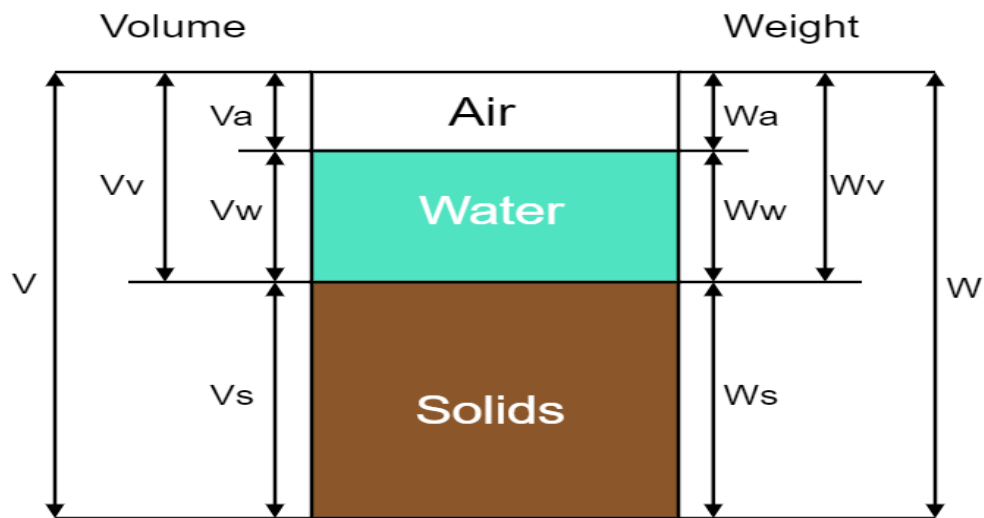


Fig.(1) three phases of soil

$$n = \frac{V_v}{V} \dots\dots\dots(4)$$

the degree of saturation (S) is defined as the ratio of the volume of water to the volume of voids, or

$$S = \frac{V_w}{V_v} \dots\dots\dots (5)$$

It is commonly expressed as a percentage.

The relationships between void ratio and porosity can be derived from Eqs.1,3,4 as follows:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1 - n} \dots\dots\dots (6)$$

Also from Eq.6,

$$n = \frac{e}{1 + e} \dots\dots\dots(7)$$

The common terms used for weight relationships are moisture content and unit weight. Moisture content ( $w$ ) is also referred to as water content and is defined as the ratio of the weight of water to the weight of solids in a given volume of soil:

$$w = \frac{W_w}{W_s} \dots\dots\dots(8)$$

Unit weight ( $\gamma$ ) is the weight of soil per unit volume. Thus,

$$\gamma = \frac{W}{V} \dots\dots\dots(9)$$

The unit weight can also be expressed in terms of the weight of soil solids, the moisture content, and the total volume. From Eqs. 2,8, and 9,

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s \left(1 + \frac{W_w}{W_s}\right)}{V} = \frac{W_s(1 + w)}{V} \dots\dots\dots(10)$$

Sometimes referred to the unit weight defined by Eq.9 as the moist unit weight.

Often, to solve earthwork problems, one must know the weight per unit volume of soil. Excluding water. This weight is referred to as the dry unit weight,  $\gamma_d$ . Thus,

$$\gamma_d = \frac{W_s}{V} \dots\dots\dots(11)$$

From Eqs.10 and 11, the relationships of unit weight, dry unit weight, and moisture content can be given as:

$$\gamma_d = \frac{\gamma}{1+W} \dots\dots\dots(12)$$

## 2-2 Mass-Volume Relationships

Sometimes it is convenient to express soil densities in terms of mass densities ( $\rho$ ). The *SI* unit of mass density is Kilograms cubic meters ( $\text{Kg}/\text{m}^3$ ). We can write the density equations (Similar to Eqs. 9 and 11) as,

$$\rho = \frac{M}{V} \dots\dots\dots(13)$$

And

$$\rho_d = \frac{M_s}{V} \dots\dots\dots(14)$$

Where:  $\rho$ : density of soil ( $\text{Kg}/\text{m}^3$ ),  $M$ : the total mass of the soil Sample ( $\text{Kg}$ ),  $M_s$ : mass of the soil solids in the sample ( $\text{Kg}$ ),  $V$ : The unit of Total volume ( $\text{m}^3$ ) the unit weight in  $\text{KN}/\text{m}^3$  can be obtained from densities in  $\text{Kg}/\text{m}^3$ .

## 2- Relationships among Unit Weight, Void Ratio, Moisture Content, and specific Gravity

To obtain a relationship among unit weight (or density), void ratio, and moisture content, let's consider a volume of soil in which the volume of the soil solids is one, as shown in Figure.2 if the volume of the soil solids is one, then the volume of voids is numerically equal to the void ratio,  $e$  (from Eq.3). The weights of soil solids and water can be given as

$$G_s = \frac{\gamma_s}{\gamma_w}$$

$$W_s = G_s \gamma_w$$

$$W_{w=w} W_{s=w} = w G_s \gamma_w$$

Where  $G_s$ : specific gravity of soil solids,  $w$ : water content,  $\gamma_w$  unit weight of water

Now, using the definition of unit weight ( $\gamma$ ) and dry unit weight ( $\gamma_d$ ) ( Eqs.9 and 11):

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + w G_s \gamma_w}{1 + e} = \frac{(1 + w) G_s \gamma_w}{1 + e} \dots\dots\dots(15)$$

and

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1 + e} \dots\dots\dots(16)$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 \dots\dots\dots (17)$$

Because the weight of water for the soil element under consideration is  $w G_s \gamma_w$  the volume occupied by water is:

$$V_w = \frac{W_w}{\gamma_w} = \frac{w G_s \gamma_w}{\gamma_w} = w G_s$$

Hence, from the definition of degree of saturation (Eq.5),

$$S = \frac{V_w}{V_v} = \frac{w G_s}{e}$$

$$S e = w G_s \dots\dots\dots(18)$$

This equation is useful for solving problems involving three-phase relationships.

If the soil sample is saturated, that is, the void space is completely filled with water (Figure3), the relationship for the saturated unit weight ( $\gamma_{sat}$ ) can be derived similarly:

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{1 + e} = \frac{(G_s + e) \gamma_w}{1 + e} \dots\dots\dots(19)$$

Also, from Eq.18 with S=1,

$$e = w G_s \dots\dots\dots(20)$$

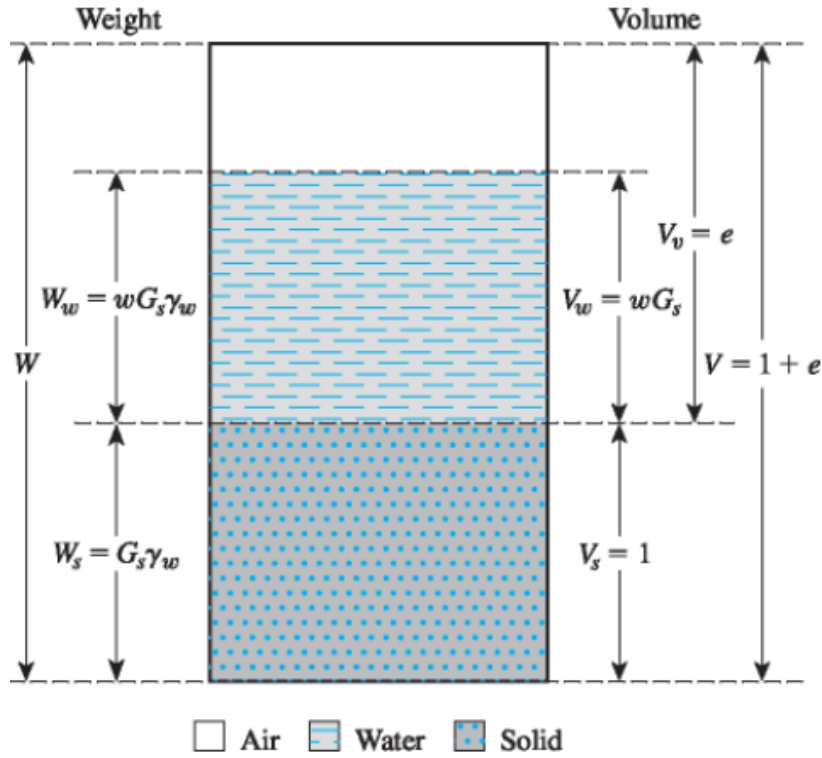
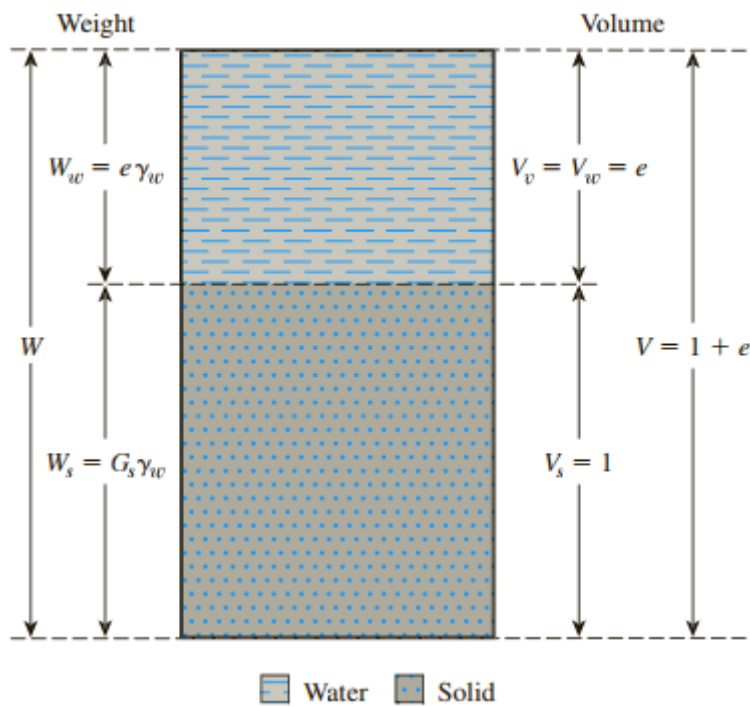


Figure (2) Three phases soil elements with volume = 1



Figure(3) Saturated soil element of  $V_s$  equal to one

As mentioned before, due to the convenience of working with densities in the SI system, the following equations, similar to unit-weight relationships given in Eqs.15,16, and 19, will be useful:

$$\text{Density}=\rho = \frac{(1+w)G_s\rho_w}{1+e} \dots\dots\dots(21)$$

$$\text{Dry density}=\rho_d = \frac{G_s\rho_w}{1+e} \dots\dots\dots(22)$$

$$\text{Saturated density}=\rho_{sat} = \frac{(G_s+e)\rho_w}{1+e} \dots\dots\dots(23)$$

Where  $\rho_w$  = density of water = 1000 Kg/m<sup>3</sup>

Equation (21) may be derived by referring to the soil element shown in Figure 4, in which the volume of soil solids is equal to 1 and the volume of voids is equal to  $e$ .

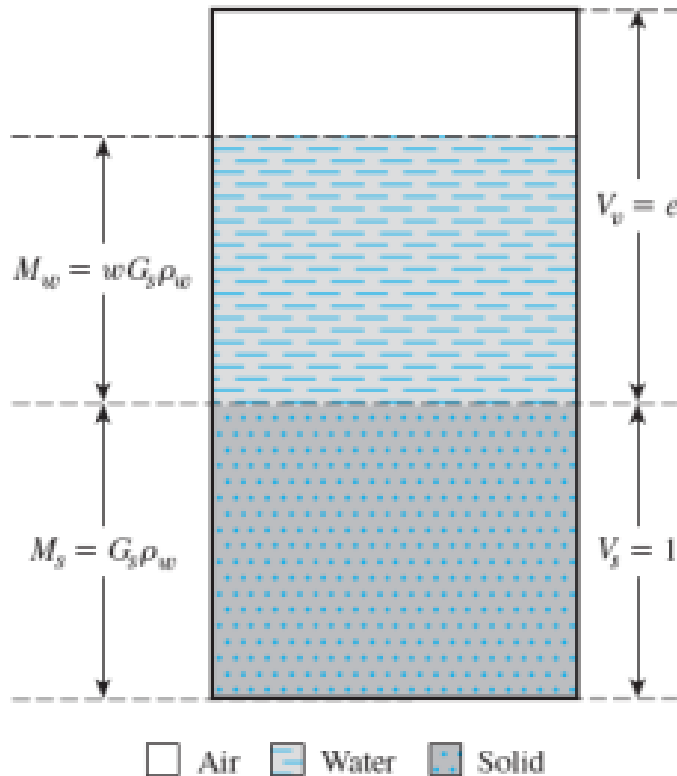


Figure (4) Three separate phases of a soil element showing mass–Volume relationship.

### 3.4 Various Unit-Weight Relationships

In Sections 3.2 and 3.3, we derived the fundamental relationships for the moist unit weight, dry unit weight, and saturated unit weight of soil. Several other forms of relationships that can be obtained for  $\gamma$ ,  $\gamma_d$ , and  $\gamma_{sat}$  are given in Table (3.1). Some typical values of void ratio, moisture content in a saturated condition, and dry unit weight for soils in a natural state are given in Table (3.2)

**Table 3.1** Various Forms of Relationships for  $\gamma$ ,  $\gamma_d$ , and  $\gamma_{sat}$

Moist unit weight ( $\gamma$ )		Dry unit weight ( $\gamma_d$ )		Saturated unit weight ( $\gamma_{sat}$ )	
Given	Relationship	Given	Relationship	Given	Relationship
$w, G_s, e$	$\frac{(1+w)G_s\gamma_w}{1+e}$	$\gamma, w$	$\frac{\gamma}{1+w}$	$G_s, e$	$\frac{(G_s+e)\gamma_w}{1+e}$
$S, G_s, e$	$\frac{(G_s+Se)\gamma_w}{1+e}$	$G_s, e$	$\frac{G_s\gamma_w}{1+e}$	$G_s, n$	$[(1-n)G_s+n]\gamma_w$
$w, G_s, S$	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	$G_s, n$	$G_s\gamma_w(1-n)$	$G_s, w_{sat}$	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
$w, G_s, n$	$G_s\gamma_w(1-n)(1+w)$	$G_s, w, S$	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	$e, w_{sat}$	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
$S, G_s, n$	$G_s\gamma_w(1-n)+nS\gamma_w$	$e, w, S$	$\frac{eS\gamma_w}{(1+e)w}$	$n, w_{sat}$	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		$\gamma_{sat}, e$	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$	$\gamma_d, e$	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
		$\gamma_{sat}, n$	$\gamma_{sat}-n\gamma_w$	$\gamma_d, n$	$\gamma_d+n\gamma_w$
		$\gamma_{sat}, G_s$	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	$\gamma_d, S$	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
				$\gamma_d, w_{sat}$	$\gamma_d(1+w_{sat})$

**Table 3.2** Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, $e$	Natural moisture content in a saturated state (%)	Dry unit weight, $\gamma_d$	
			lb/ft <sup>3</sup>	kN/m <sup>3</sup>
Loose uniform sand	0.8	30	92	14.5
Dense uniform sand	0.45	16	115	18
Loose angular-grained silty sand	0.65	25	102	16
Dense angular-grained silty sand	0.4	15	121	19
Stiff clay	0.6	21	108	17
Soft clay	0.9–1.4	30–50	73–93	11.5–14.5
Loess	0.9	25	86	13.5
Soft organic clay	2.5–3.2	90–120	38–51	6–8
Glacial till	0.3	10	134	21

