وزارة التعليم العالي والبحث العلمي
جامعة الفرات الاوسط التقنية
المعهد التقني / بابل
قسم التقنيات المدنية

ENGINEERING MECHANICS

محاضرات الميكانيك الهندسي – المرحلة الاولى

العام الدراسي 2023/2022

اعداد
م. زهير ظاهر حبيب

ENGINEERING MECHANICS

References:

1-Engineering Mechanics — A .Higdon
2-Mechanics for Engineers — F.P Beer

Mechanics: is that branch of physical sciences which describes the motion of bodies with rest being considered a special case of motion .

Mechanics of rigid bodies: is divided into two portions:

1-Statics: deals with bodies at rest
2-Dynamics: deals with bodies in motion

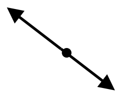
Physical Quantities: is classified to:

1-Scalar quantities: have only magnitude (mass, volume)
2-Vector quantities: have both magnitude and direction (couple, force)

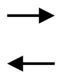
FORCE: any action which change or try to change the shape, volume or the motion of a body .

Classification of forces:

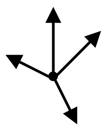
1- Collinear



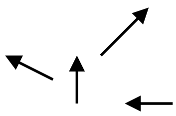
2- Parallel forces



3- Concurrent forces



4- Non parallel, non-concurrent forces



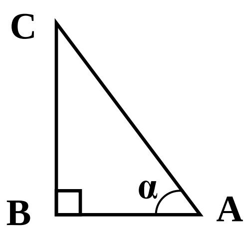
Right angle triangle:

$\sin \alpha = BC / AC \Rightarrow BC = AC \sin \alpha$

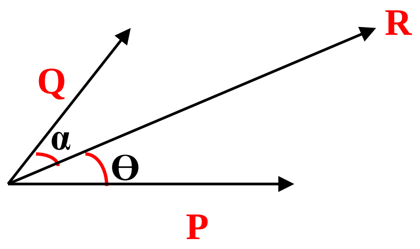
$\cos \alpha = AB / AC \Rightarrow AB = AC \cos \alpha$

$\tan \alpha = BC / AB$

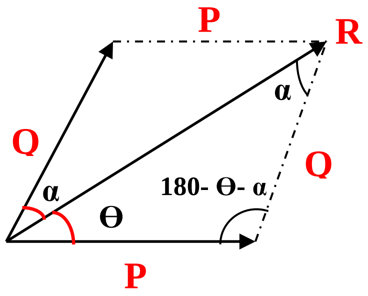
$(AC)^2 = (AB)^2 + (BC)^2$



Parallelogram:

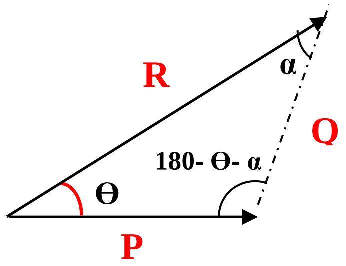


Cos. Law:



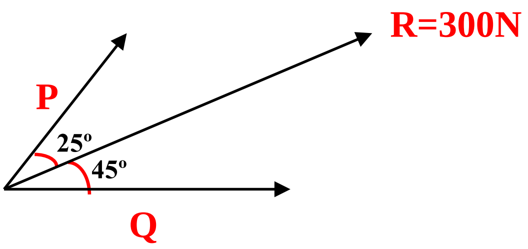
$$R^2 = P^2 + Q^2 - 2PQCos(180 - \theta - \alpha)$$

Sin. Law:

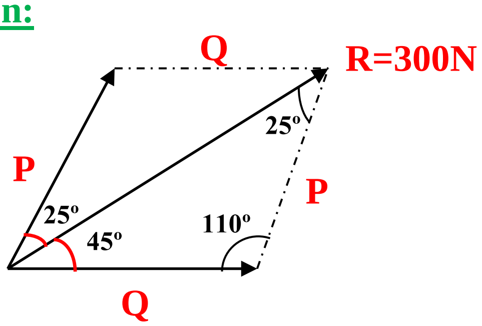


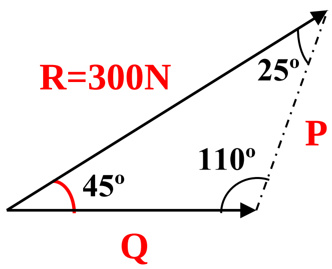
$$\frac{R}{Sin(180-\theta-\alpha)} = \frac{Q}{Sin\theta} = \frac{P}{Sin\alpha}$$

Example: **Resolve the (300N) force into two components as shown in figure.**



Solution:





$(180-\theta-\alpha)$ = 180-45-25 =110

$$\frac{R}{Sin(180-\theta-\alpha)}=\frac{Q}{Sin\theta}=\frac{P}{Sin\alpha}$$

$$\frac{300}{Sin110}=\frac{Q}{Sin25°}=\frac{P}{Sin45°}$$

$$\frac{300}{Sin110}=\frac{Q}{Sin25°}$$

$$300Sin25°=QSin110°$$

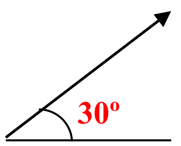
$$Q=134.92\ N$$

$$\frac{300}{Sin110}=\frac{P}{Sin45°}$$

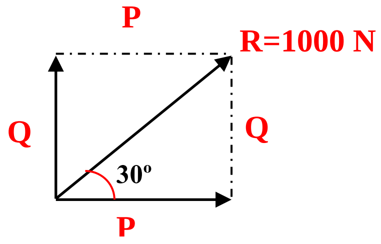
$$300Sin45°=PSin110°$$

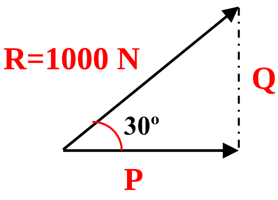
$$P=225.74N$$

Example: **Resolve the (1000N) force shown in figure into two Perpendicular components.**



Solution:

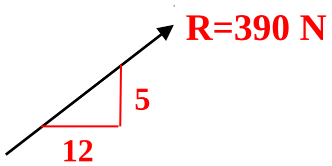




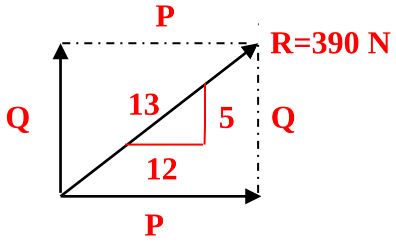
$\sin 30°=\frac{Q}{1000}$ $\qquad Q=1000\sin 30°=500N$

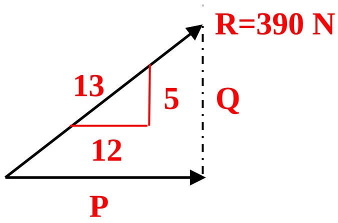
$\cos 30°=\frac{P}{1000}$ $\qquad P=1000\ \cos 30°\ =866N$

Example: Resolve the (390N) force shown in figure into two Perpendiculars components.



Solution





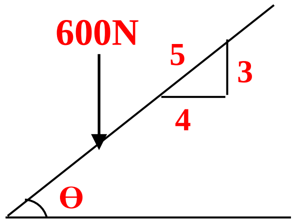
$$\frac{5}{13} = \frac{Q}{390}$$

$$Q = 150N$$

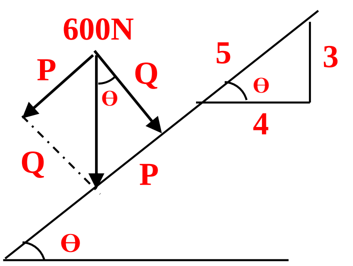
$$\frac{12}{13} = \frac{P}{390}$$

$$P = 360N$$

Example: Resolve the (600N) force shown in figure into two Components one of them perpendicular on the inclined surface and the another parallel to it



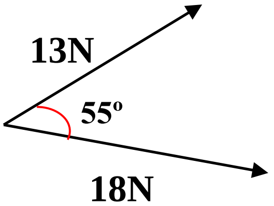
Solution:



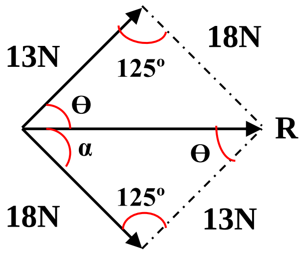
$\frac{3}{5} = \frac{P}{600} \quad \Longrightarrow \quad 3 \times 600 = 5 \times P \quad \Longrightarrow \quad P = 360N$

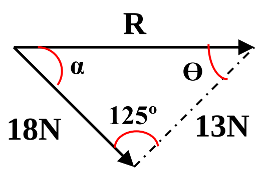
$\frac{4}{5} = \frac{Q}{600} \quad \Longrightarrow \quad 4 \times 600 = 5 \times Q \quad \Longrightarrow \quad Q = 480N$

Example: Determine the magnitude and direction of resultant of the two forces shown in figure .



Solution:





$$180 - \theta - \alpha = 180 - (\theta + \alpha) = 180 - 55 = 125°$$
$$R^2 = P^2 + Q^2 - 2PQCos(180 - \theta - \alpha)$$
$$R^2 = (13)^2 + (18)^2 - 2 \times 13 \times 18 \times Cos125°$$
$$R^2 = 761.43N$$
$$R = 27.59N$$

$$\frac{R}{Sin(180 - \theta - \alpha)} = \frac{P}{Sin\alpha}$$

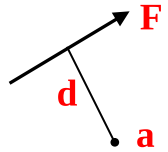
$$\frac{27.59}{Sin125°} = \frac{13}{Sin\alpha}$$

$$13 \times Sin125° = 27.59 \times Sin\alpha$$
$$Sin\alpha = 0.385$$
$$\alpha = \sin^{-1} 0.385 = 22.7°$$
$$\theta = 55 - 22.7 = 32.3°$$

Moment of Force : Is a measure to its tendency to turn a force about a point or axis.

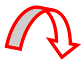
Mathematical expression of moment:

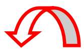
$$Ma = F.\ d$$



F = the magnitude of force.
d = moment arm = the perpendicular distance between the force and the point.

Direction of Moment:

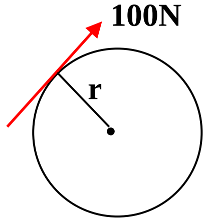
Clock wise - 

Counter clockwise + 

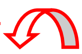
Units of Moment: N.cm, N.m , KN.m , Ib.in .

Varignan's Theory: the moment of a force about any point or axis is equal to the vector sum of the moments of its components about the same point or axis .

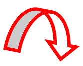
Example: Determine the moment of the (100N) force shown in figure about the axis through point A , if (r=50cm).



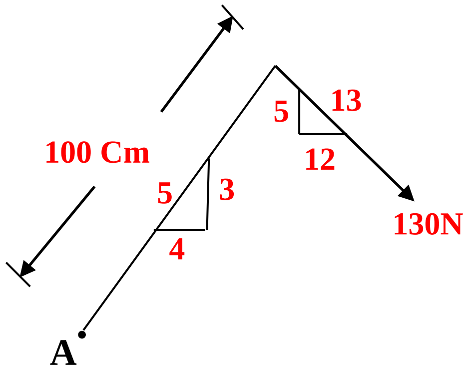
Solution:

+  Ma = F. d

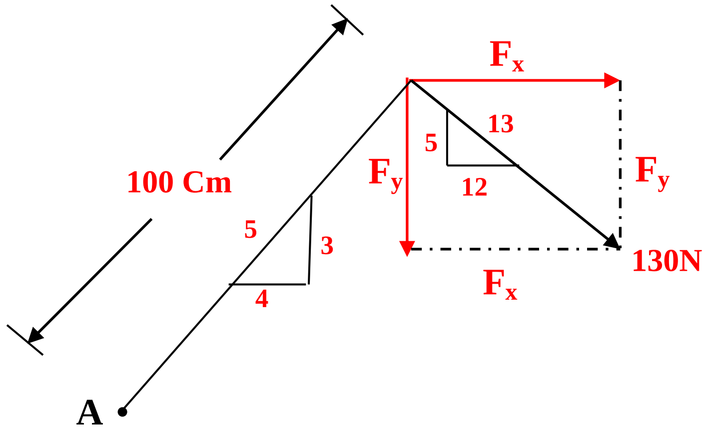
= - 100 × 50= - 5000 N.cm

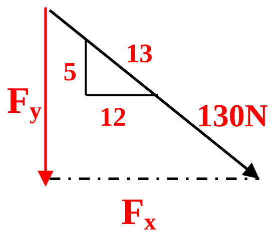
= 5000N.cm 

Example: **Determine the moment of the (130N) force shown in figure about the axis through point A .**



Solution:



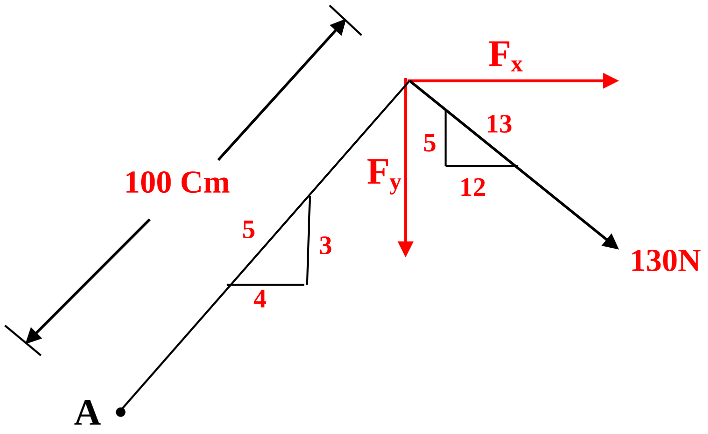


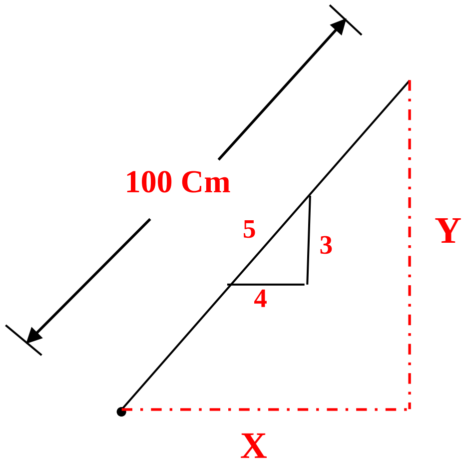
$$\frac{Fx}{130} = \frac{12}{13}$$

$$Fx \times 13 = 130 \times 12 \quad \Rightarrow \quad Fx = \frac{130 \times 12}{13} = 120N$$

$$\frac{Fy}{130} = \frac{5}{13}$$

$$Fy \times 13 = 130 \times 5 \quad \Rightarrow \quad Fy = \frac{130 \times 5}{13} = 50N$$





$\frac{X}{100} = \frac{4}{5} \Longrightarrow 5X = 400 \Longrightarrow X = 80\ \text{cm}$

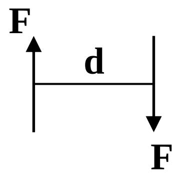
$\frac{Y}{100} = \frac{3}{5} \Longrightarrow 5Y = 300 \Longrightarrow Y = 60\ \text{cm}$

$$+\ Ma = -Fx * Y - Fy * X$$
$$= -120 * 60 - 50 * 80$$
$$= -11200\ \text{N.cm}$$
$$= 11200\ \text{N.cm}$$

Couples

A couple consists of two equal forces which have parallel line of actions and apposite senses and work on turn the body .

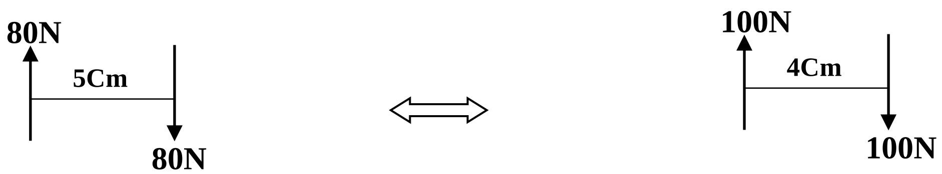
Moment of a couple: Mc



$$Mc = F.d$$

Mc: the sum of the moments of the forces.
F : the magnitude of the force.
d : the perpendicular distance between the forces .

Transformation of a couple:

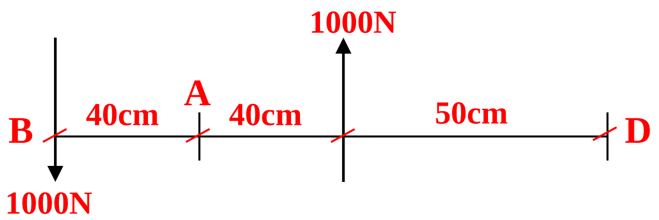


$+\ Mc = F.d = -80 \times 5$
$= -400\ N.cm = 400N.cm$

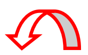
$Mc = F.d = -100 \times 4$
$= -400\ N.cm = 400N.cm$

NOTE: the moment of a couple about any point is equal.

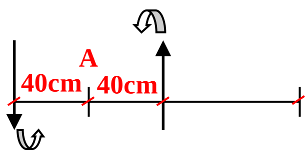
Example: Determine the moment of the couple shown in figure about the axis through points A, B, D.

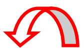


Solution:

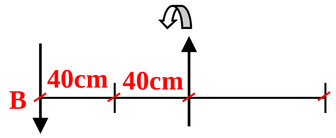


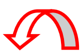
$+ \curvearrowleft \quad Mc_{(A)} = 1000\times 40 + 1000\times 40 = 80000\ N.cm$



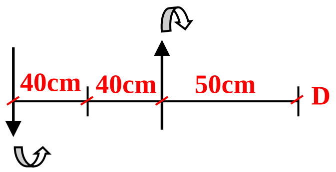


$+ \curvearrowleft \quad Mc_{(B)} = 1000\times (40+40) = 80000\ N.cm$



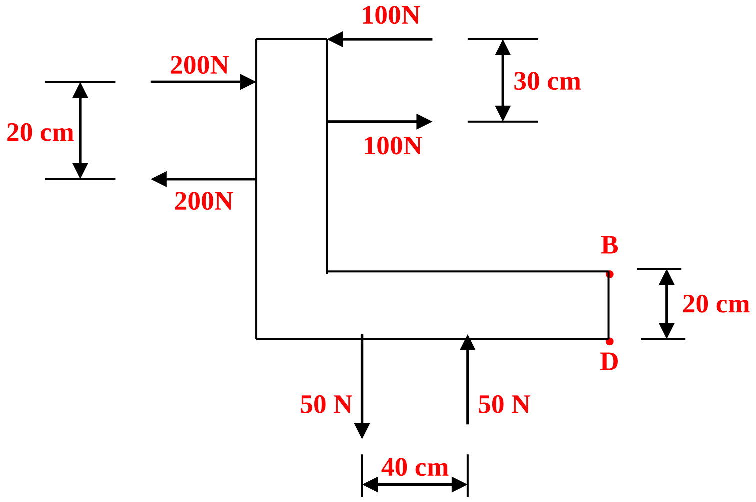


$+ \curvearrowleft \quad Mc_{(D)} = 1000\times (40+40+50) - 1000\times 50 = 80000\ N.Cm$

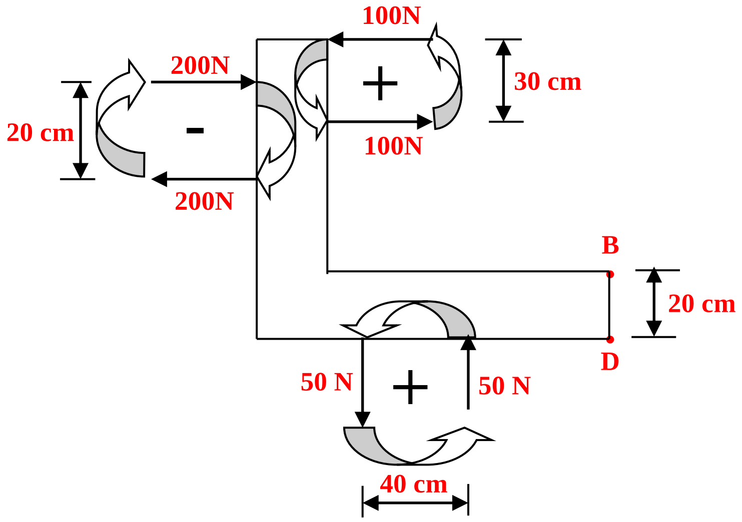


NOTE: two or more couples may be replaced by a single couple have the same magnitude and direction of moment results by the summation of moments of the original couples .

Example: Replace the following couples shown in figure by a single couple its forces effects horizontally at points B, D.

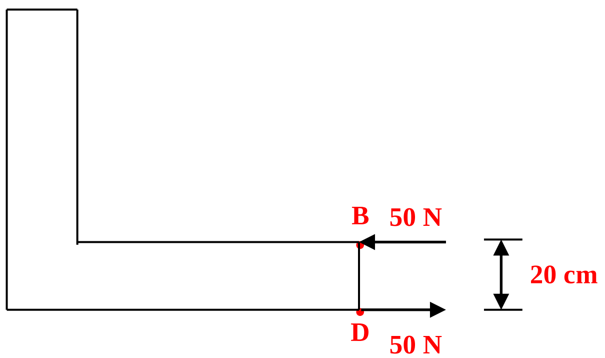


Solution:



$$+\curvearrowleft \quad Mc = 100 \times 30 + 50 \times 40 - 200 \times 20$$
$$= 1000\ N.cm$$

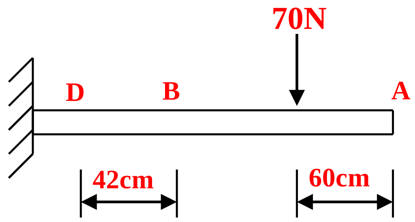
$$Mc = F.d$$
$$1000 = F \times 20$$
$$F = 50N$$



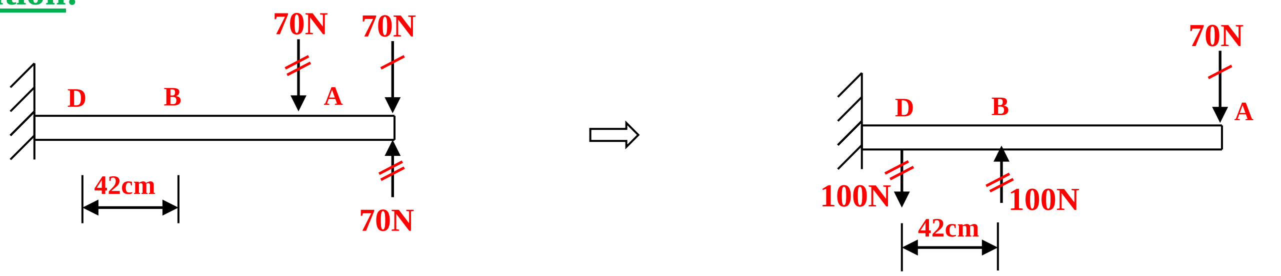
Resolution of a force into a force and a couple :

A force can be replaced by a parallel force at any different point and a couple by addition of two equal collinear forces of opposite senses to the force system.

Example: Replace the (70N) force shown in figure by a force which acts at point (A) and a couple whose forces act vertically at points (B, D) .



Solution:

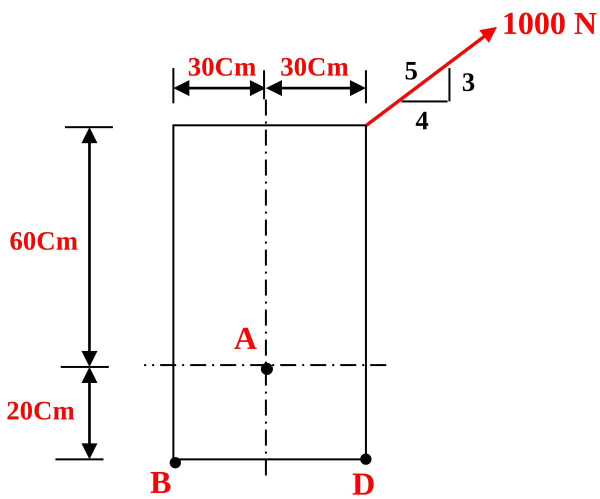


$$+ \circlearrowleft M_c = F.d = 70 \times 60 = 4200\ N.cm$$

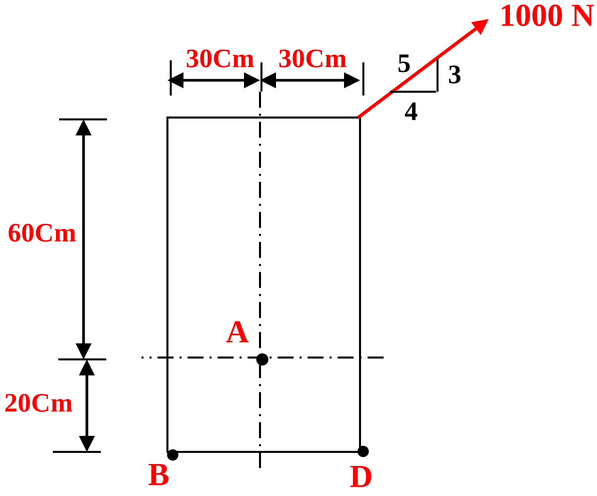
$$M_c = F.d$$

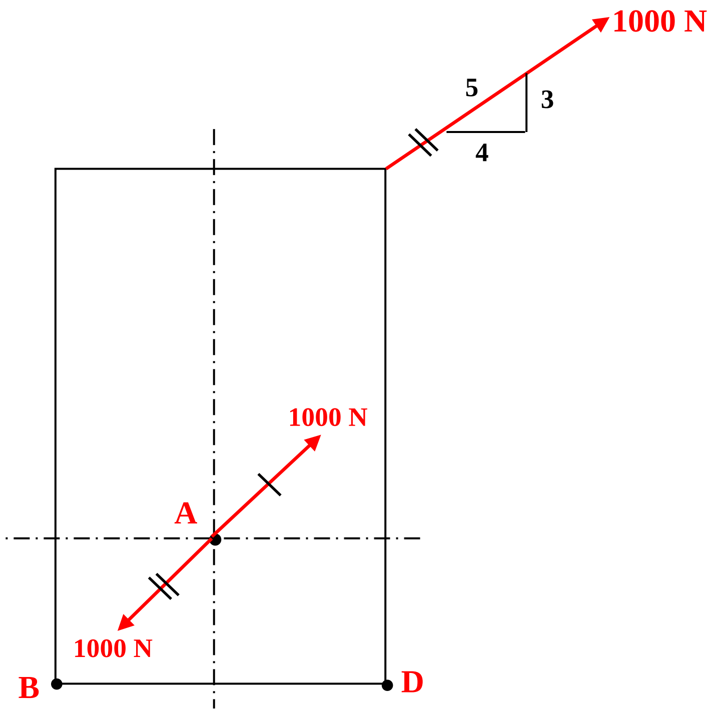
$$4200 = F \times 42 \quad \Rightarrow \quad F = 100\ N$$

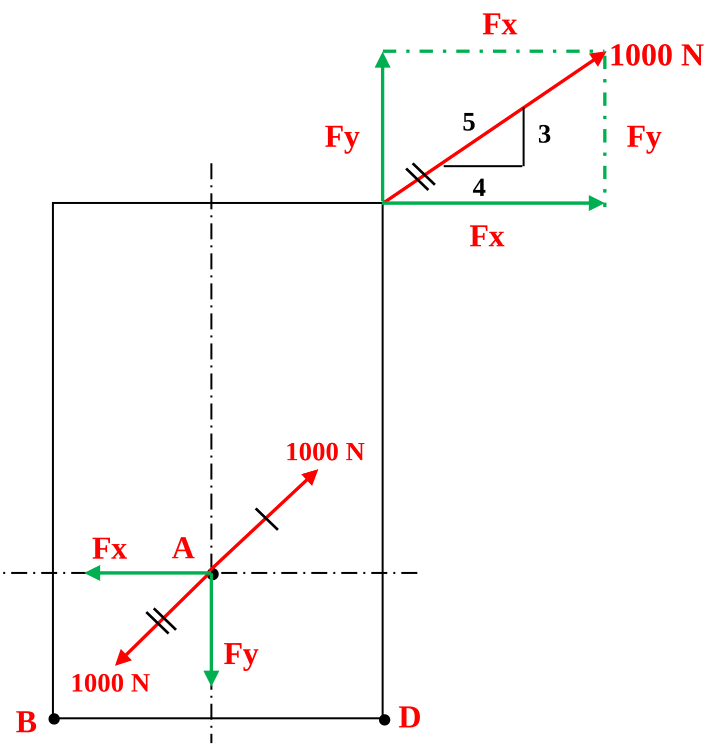
Example: Replace the (1000N) force shown in figure by a force which acts at point (A) and a couple whose forces act vertically at points (B, D) .



Solution:







$$\frac{Fx}{1000} = \frac{4}{5}$$

$$Fx * 5 = 4 * 1000 \quad \Rightarrow \quad Fx = 800\ N$$

$$\frac{Fy}{1000} = \frac{3}{5}$$

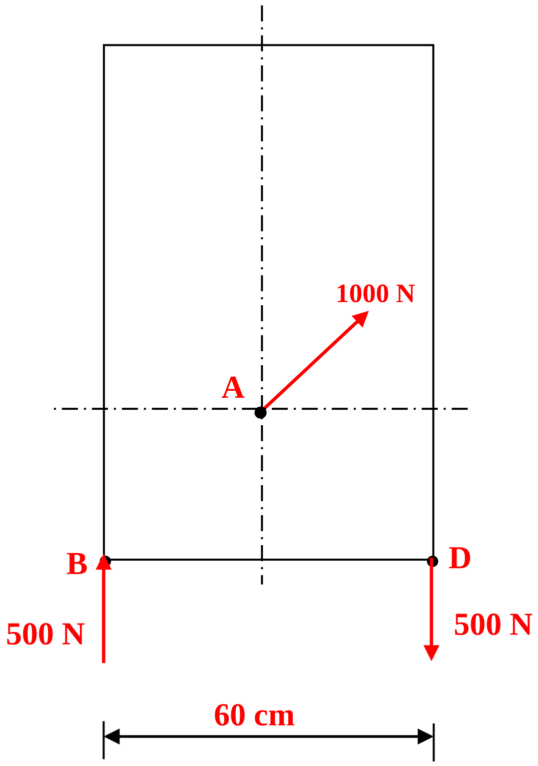
$$Fy * 5 = 3 * 1000 \quad \Rightarrow \quad Fy = 600\ N$$

$$+\circlearrowleft \quad Mc = -800 \times 60 + 600 \times 30 = -30000\ N.cm$$

$$= 30000\ N.cm \ \circlearrowright$$

$$Mc = F \times d$$

$$30000 = F \times 60 \quad \Rightarrow \quad F = 500\ N$$

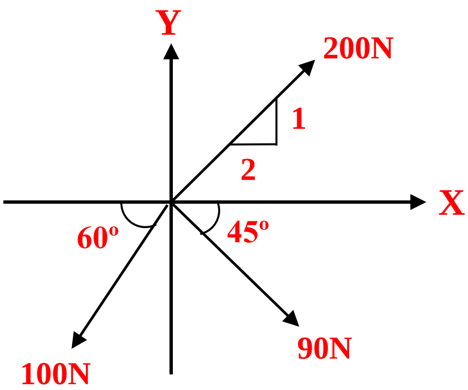


Resultant: the resultant is the simplest force which can replace the original force system without changing its external effect on the body.

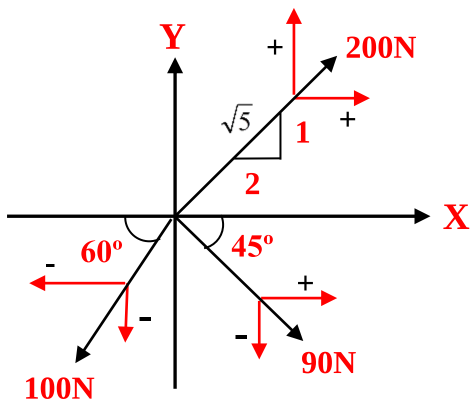
If R=0 the body is in equilibrium.
If R≠0 the body will be accelerated.

(1): Resultant of concurrent forces: expected resultant is a force.

Example: Determine the magnitude and direction of the resultant for the force system shown in figure.



Solution:



$$+\rightarrow Rx = 200 \times \frac{2}{\sqrt{5}} + 90\cos 45 - 100\cos 60$$
$$= 178.88 + 63.63 - 50$$
$$= 192.51\ N \rightarrow$$

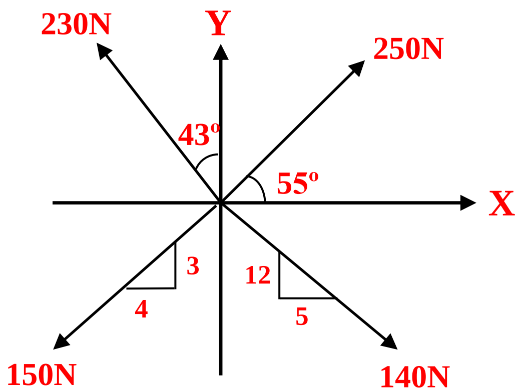
$$+\uparrow Ry = 200 \times \frac{1}{\sqrt{5}} - 90\sin 45 - 100\sin 60$$
$$= 89.44 - 63.63 - 86.6$$
$$= -60.79\ N = 60.79N \downarrow$$

$$R = \sqrt{Rx^2 + Ry^2}$$
$$R = \sqrt{(192.51)^2 + (60.79)^2}$$
$$= 201.87N$$

$$\tan\theta = \frac{Ry}{Rx} = \frac{60.79}{192.51} = 0.315$$

$$\theta = \tan^{-1} 0.315 = 17.5°$$

Example: Determine the magnitude and direction of the resultant for the force system shown in figure.



Solution:

$$\overset{+}{\longrightarrow} Rx = 250\cos 55° + 140 \times \frac{5}{13} - 150 \times \frac{4}{5} - 230\sin 43°$$

$$= 143.39 + 53.84 - 120 - 156.85$$

$$= -79.62N = 79.62N \longleftarrow$$

$$+\uparrow Ry = 250\sin 55° - 140 \times \frac{12}{13} - 150 \times \frac{3}{5} + 230\cos 43°$$

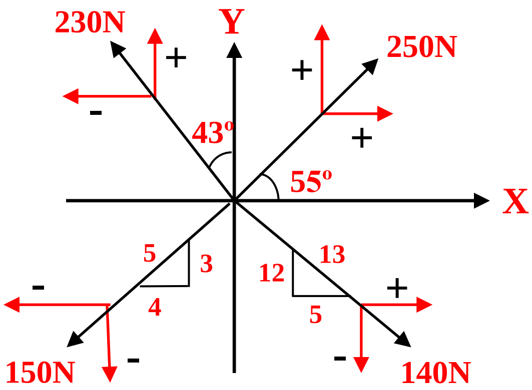
$$= 204.78 - 129.23 - 90 + 168.21$$

$$= 153.76N$$

$$R = \sqrt{Rx^2 + Ry^2}$$

$$R = \sqrt{(79.62)^2 + (153.76)^2}$$

$$= 173.16N$$



$$\tan\theta = \frac{Ry}{Rx} = \frac{153.76}{79.62} = 1.931$$

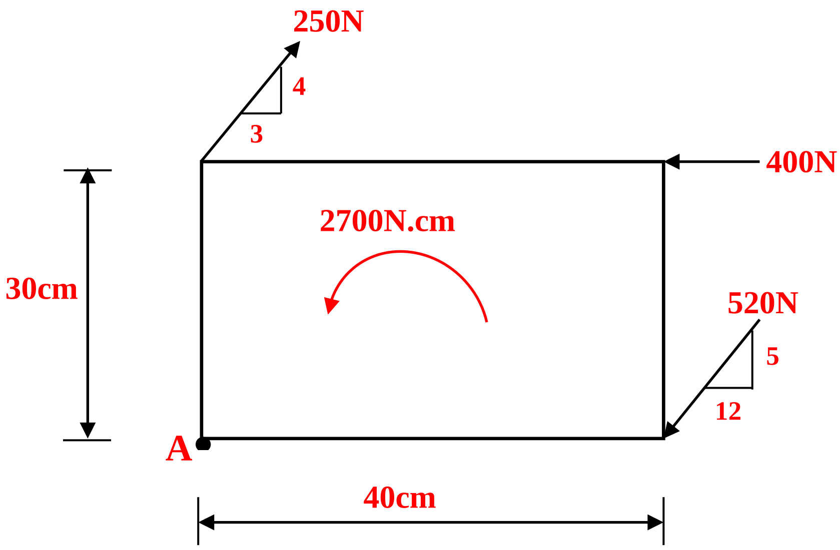
$$\theta = \tan^{-1} 1.931 = 62.62°$$

(2) : Resultant of non-concurrent, non-parallel forces:

If $R \neq 0$ the resultant is a force

If $R = 0$ the resultant is a couple and $Mc = \sum Mo$

Example: Determine the resultant of the forces and the couple shown in figure and locate it with respect to point (A).



Solution:

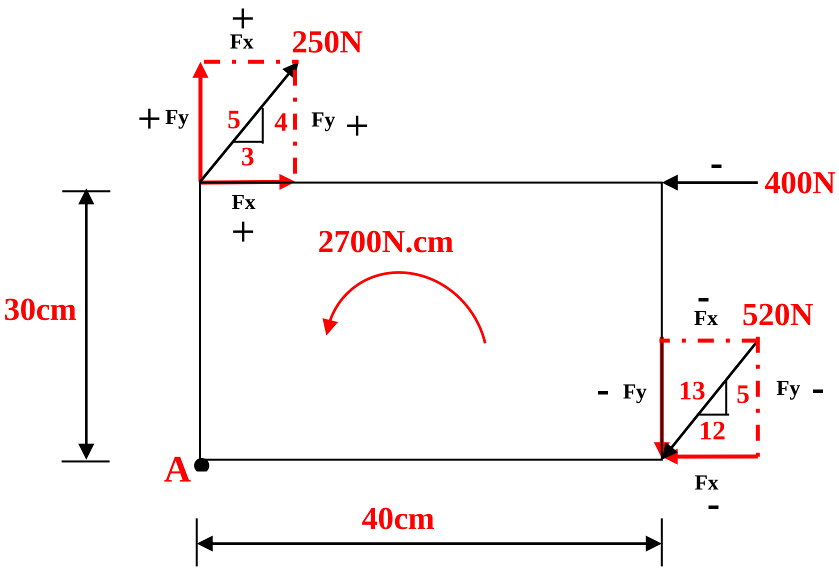
$$+ \longrightarrow Rx = 250 \times \frac{3}{5} - 520 \times \frac{12}{13} - 400$$

$$= -730N = 730N \longleftarrow$$

$$+\uparrow \quad Ry = 250 \times \frac{4}{5} - 520 \times \frac{5}{13} = 0$$

$$R = \sqrt{(Rx)^2 + (Ry)^2}$$

$$= \sqrt{(730)^2 + (0)^2} = 730N$$



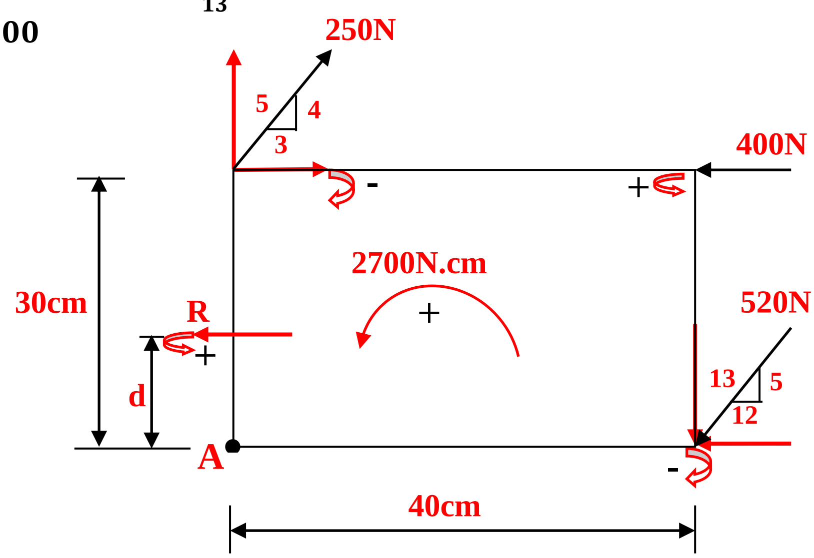
$$R \times d = \sum MA$$

$$730 \times d = 400 \times 30 + 2700 - 250 \times \frac{3}{5} \times 30 - 520 \times \frac{5}{13} \times 40$$

$$730 \times d = 12000 + 2700 - 4500 - 8000$$

$$730 \times d = 2200$$

$$d = \frac{2200}{730} = 3.01cm$$

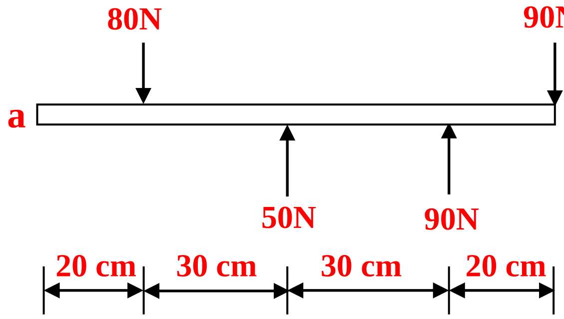


(3): Resultant of parallel force system:

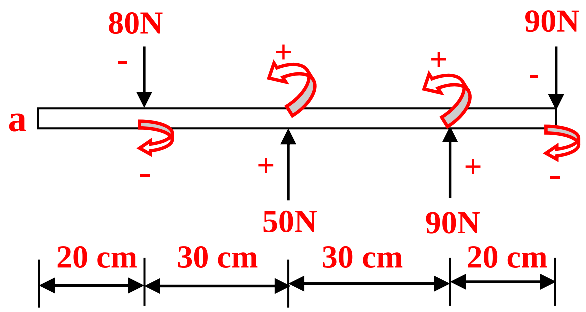
If $R \neq 0$ then the resultant is a force

If $R = 0$ then the resultant is a couple and $Mc = \sum Ma$

Example: Determine the resultant of the parallel forces shown in figure, and its location from point (a) .



Solution:



$+\uparrow \quad R = \sum Fy$

$= 50 + 90 - 80 - 90$

$= -30N = 30N \downarrow$

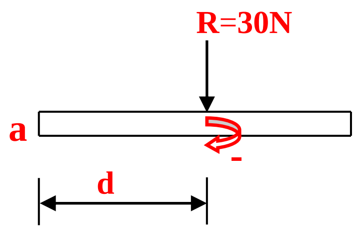
$+\circlearrowleft \quad R \times d = \sum Ma$

$-30 \times d = 50 \times 50 + 90 \times 80 - 80 \times 20 - 90 \times 100$

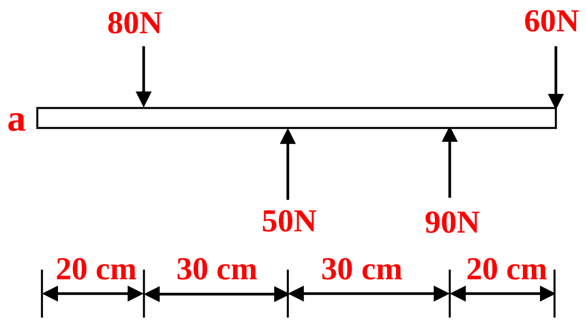
$-30 \times d = 2500 + 7200 - 1600 - 9000$

$-30 \times d = -900$

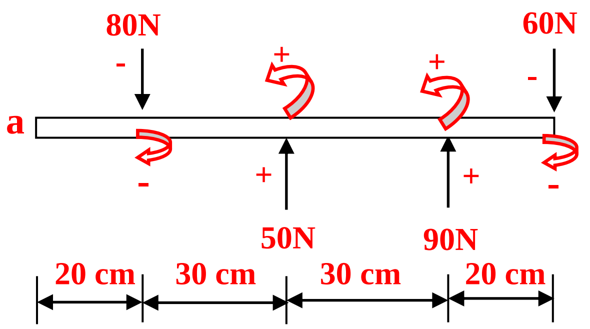
$d = \frac{-900}{-30} = 30cm$



Example: Determine the resultant of the parallel forces shown in figure, and its location from point (a) .



Solution:



$$+\uparrow \quad \boldsymbol{R = \sum Fy}$$
$$\boldsymbol{= 50 + 90 - 80 - 60}$$
$$\boldsymbol{= 0}$$

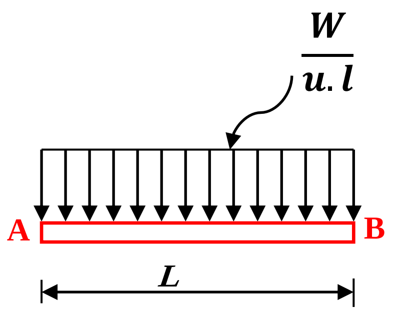
The resultant may be a couple

$$+ \circlearrowleft \quad \boldsymbol{Mc = \sum Ma}$$
$$\boldsymbol{= 50 \times 50 + 90 \times 80 - 80 \times 20 - 60 \times 100}$$
$$\boldsymbol{= 2100 N.cm}$$

DISTRIBUTED LOADS:

(1): Uniformly Distributed Loads or rectangular loads

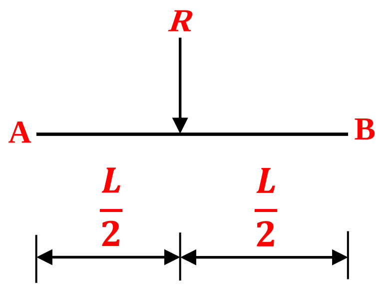
$$R = \frac{W}{u.l} \times L$$



R: resultant of the total weight of construction

$\frac{W}{u.l}$: the weight for unit length

L: the length of construction

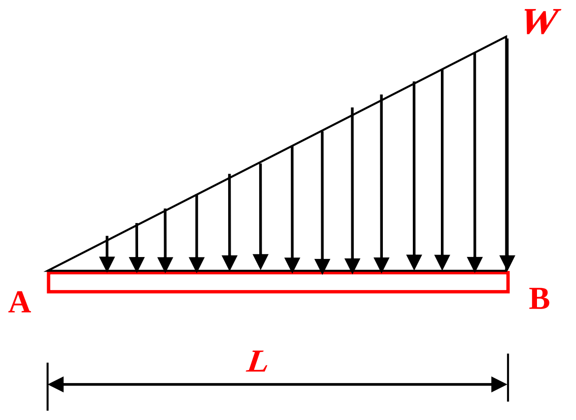


NOTE: the location of (***R***) is in the middle

i.e. $\frac{L}{2}$ from A and B

(2): Varying Loads or triangular loads

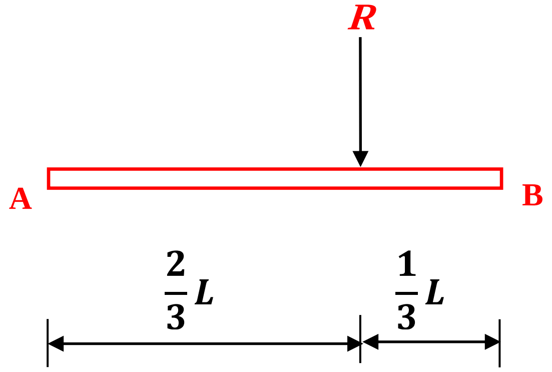
$$R = \frac{1}{2} \times W \times L$$



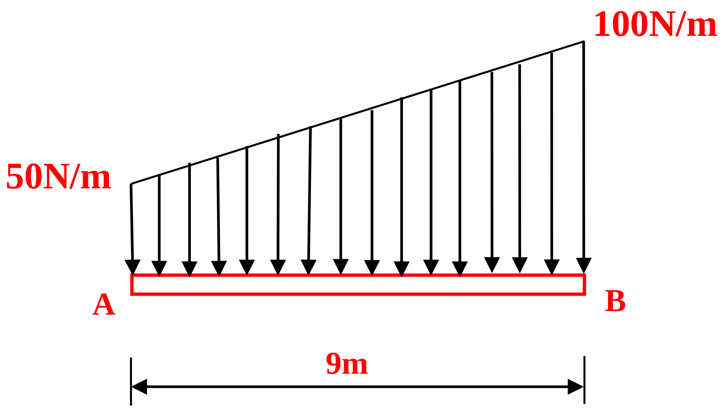
NOTE: the location of (***R***) is:

$\frac{1}{3}L$ from point B and

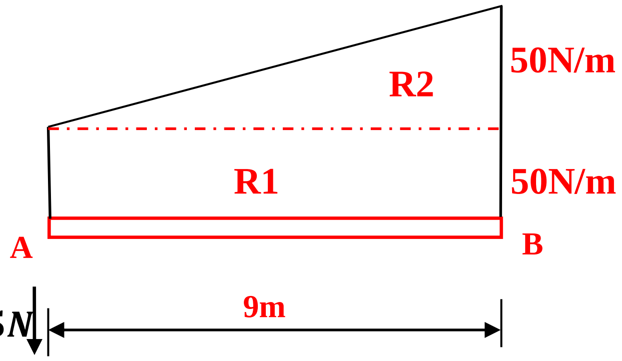
$\frac{2}{3}L$ from point A



Example: Determine the resultant of the distributed loads shown in figure and indicate its location from point (A) .



Solution:



$$+\uparrow R1 = -(9 \times 50) = -450N = 450N\downarrow$$

$$+\uparrow R2 = -\left(\frac{1}{2} \times 9 \times 50\right) = -225N = 225N\downarrow$$

$$+\uparrow R = \sum Fy = (-450) + (-225) = -675N = 675N\downarrow$$

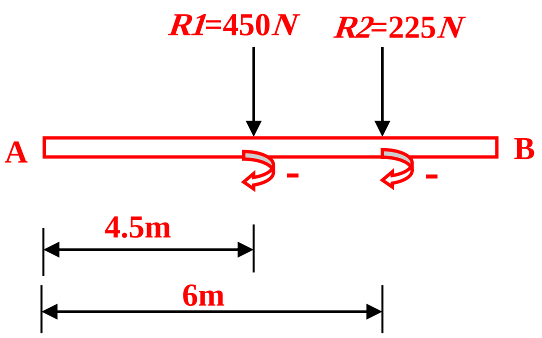
$$+\circlearrowleft \; R \times d = \sum Ma$$

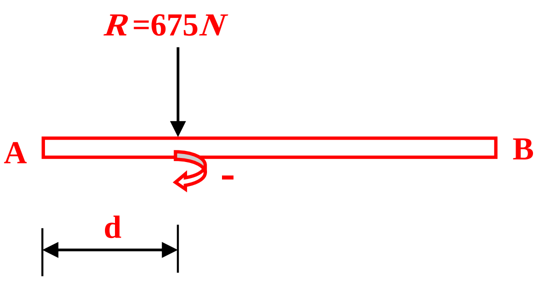
$$-675 \times d = (-450 \times 4.5) + (-225 \times 6)$$

$$-675 \times d = (-2025) + (-1350)$$

$$-675 \times d = -3375$$

$$d = \frac{-3375}{-675} = 5m$$





EQUILIBRIUM

Is the condition of the body when the resultant of forces acting on it is equal to (ZERO)

1 : Equilibrium of concurrent forces:

The resultant of this system is a force can be calculated by:

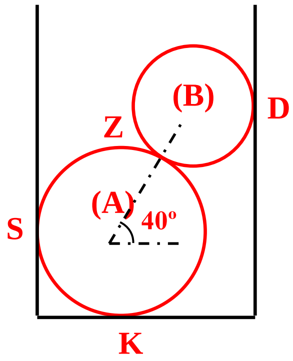
$$R = \sqrt{Rx^2 + Ry^2}$$

In equilibrium condition R=0 then:

$$Rx = \sum Fx = 0 \quad \ldots \ldots (1)$$

$$Ry = \sum Fy = 0 \quad \ldots \ldots (2)$$

Example: Find all forces which effects on the cylinder (A) shown in figure if all concurrent surfaces are smooth, and the weight of cylinder (A) is (500N), and cylinder (B) is (300N) .



Solution:

From F.B.D of cylinder (B):

$$+\uparrow \sum Fy = 0$$

$$Fz \sin 40° - 300 = 0$$

$$Fz = \frac{300}{\sin 40°} = 466.7N$$

From F.B.D of cylinder (A):

$$+\rightarrow \sum Fx = 0$$

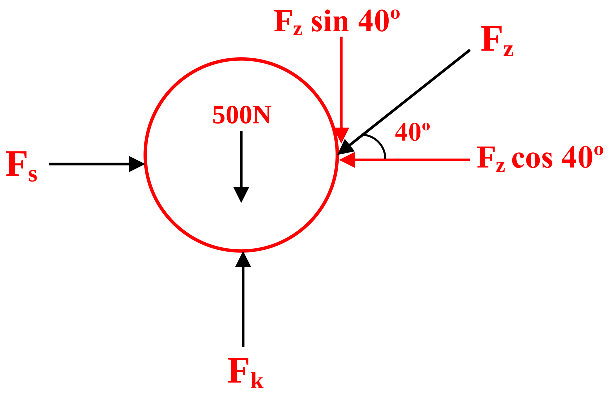
$$Fs - 466.71 \cos 40° = 0$$

$$Fs = 466.71 \cos 40° = 357.52N$$

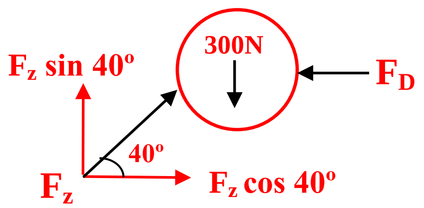
$$+\uparrow \sum Fy = 0$$

$$Fk - 500 - 466.71 \sin 40° = 0$$

$$Fk = 500 + 466.71 \sin 40° = 800N$$

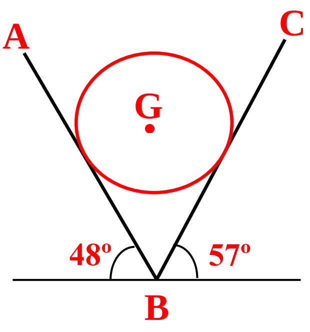


F.B.D of cylinder (A)



F.B.D of cylinder (B)

Example: Determine the forces exerted by the planes (AB) and (BC) on the cylinder shown in figure if the weight of cylinder is (900N) and the surfaces are smooth.



Solution:

$$+\rightarrow \sum Fx = 0$$

$$FAB \sin 48° - FBC \sin 57° = 0 \quad \ldots\ldots\ldots (1) \quad \times \cos 57$$

$$+\uparrow \sum Fy = 0$$

$$FAB \cos 48° + FBC \cos 57° - 900 = 0 \quad \ldots\ldots\ldots (2) \quad \times \sin 57$$

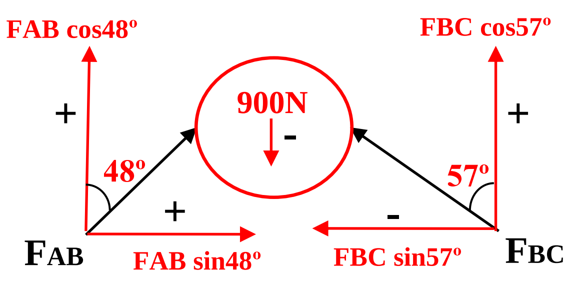
$$FAB \sin 48° \cos 57° - FBC \sin 57° \cos 57° = 0 \quad \ldots\ldots\ldots (1)$$

$$FAB \cos 48° \sin 57° + FBC \sin 57° \cos 57° - 900 \sin 57° = 0 \quad \ldots\ldots\ldots (2)$$

$$+ \overline{}$$

$$FAB(\sin 48° \cos 57° + \cos 48° \sin 57°) = 900 \sin 57°$$

$$FAB = \frac{900 \sin 57°}{\sin 48° \cos 57° + \cos 48° \sin 57°} = 781.43N$$



Substitute in equation (1) :

$$781.43 \sin 48 - FBC \sin 57 = 0$$

$$781.43 \sin 48 = FBC \sin 57$$

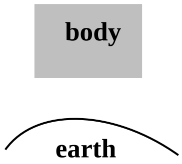
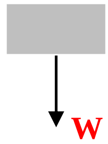
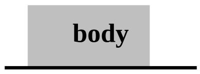
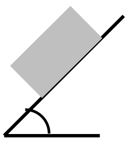
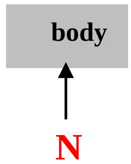
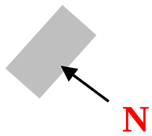
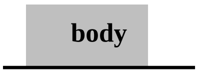
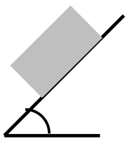
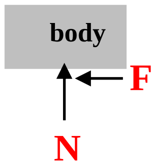
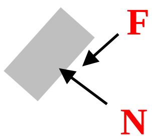
$$FBC = \frac{781.43 \sin 48}{\sin 57} = 692.42$$

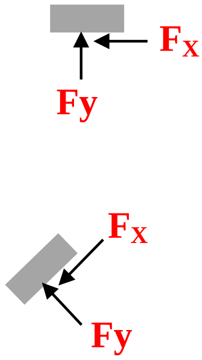
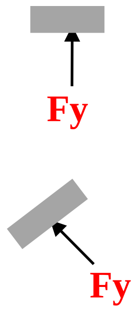
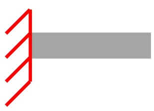
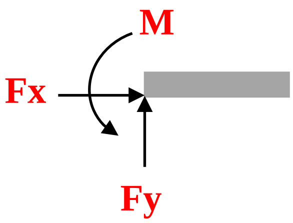
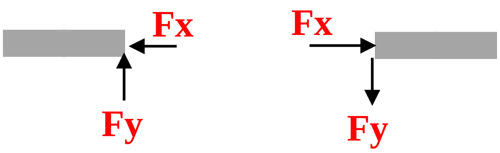
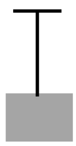
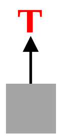
F.B.D of cylinder

<u>Free Body Diagram</u>: F.B.D

Is a diagram shown all the forces acting on the body.

<u>Types of supports</u>:

Type of support	Body diagram	F.B.D
1-Earth		
2- Smooth surface	 Plane  Inclined	 
3-Rough surface	 Plane  Inclined	 

4-Hinge		
5-Roller		
6-Fixed		
7-Internal hinge		
8-Cable		

② : Equilibrium of non concurrent forces:

The resultant of this system is:

A force can be calculated by $R = \sqrt{Rx^2 + Ry^2}$ when R≠ 0

OR A couple can be calculated by $Mc = \sum M$ when R= 0

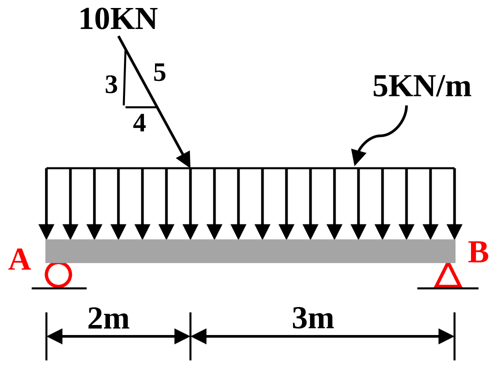
In equilibrium condition R=0 and Mc=0 then:

$Rx = \sum Fx = 0$ ....(1)

$Ry = \sum Fy = 0$ ....(2)

$Mc = \sum M = 0$ ....(3)

Example: Determine the reactions at supports (A) and (B) for the beam loaded as shown in figure .



Solution:

$+\uparrow$ $R = -(5 \times 5) = -25KN = 25KN \downarrow$

$+\rightarrow$ $Fx = 10 \times \frac{4}{5} = 8KN$

$+\uparrow$ $Fy = -(10 \times \frac{3}{5}) = -6KN = 6KN \downarrow$

$+\rightarrow$ $\sum Fx = 0$

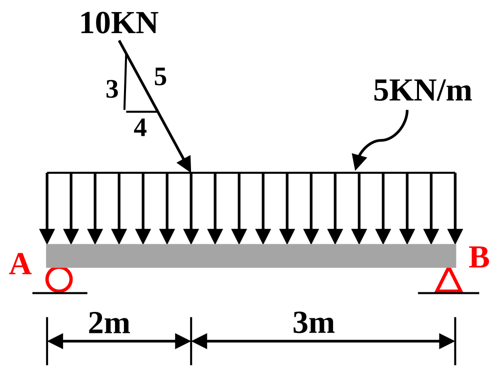
$8 - Bx = 0 \Rightarrow Bx = 8KN$

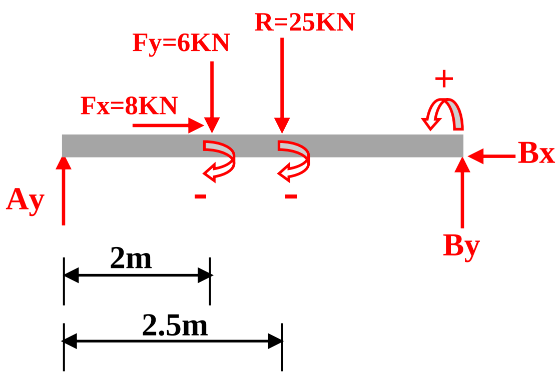
$+ \circlearrowleft \sum M_A = 0$

$By \times 5 - 6 \times 2 - 25 \times 2.5 = 0 \Rightarrow By = 14.9KN$

$+\uparrow$ $\sum Fy = 0$

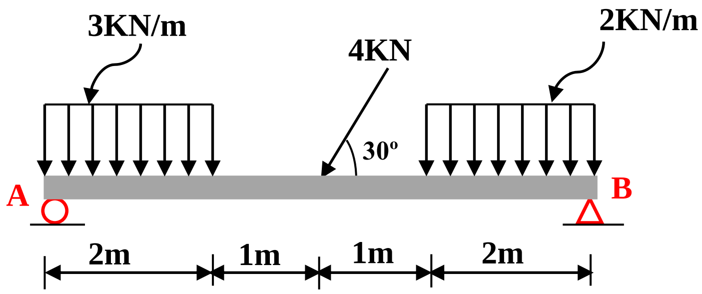
$Ay + 14.9 - 25 - 6 = 0 \Rightarrow Ay = 16.1\ KN$





F.B.D of the beam

Example: Determine the reactions at supports (A) and (B) for the beam loaded as shown in figure .



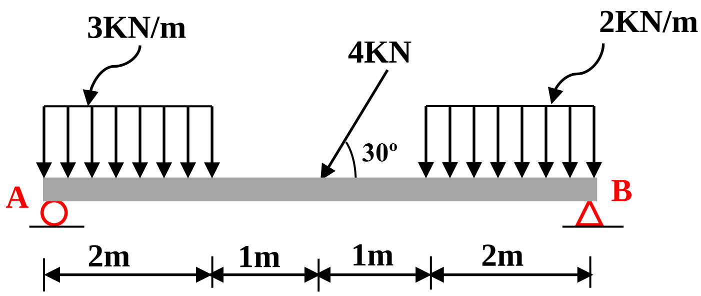
Solution:

+↑ R1 = - (3×2) = - 6KN = 6KN↓

+↑ R2 = - (2× 2) = - 4KN = 4KN↓

+→ Fx = -4Cos30 = -3.46KN = 3.46KN ←

+↑ Fy = - 4Sin30 = - 2KN = 2KN↓



+→ $\sum Fx = 0$

Bx -3.46 = 0 ⇨ **Bx = 3.46KN**

+↺ $\sum MA = 0$

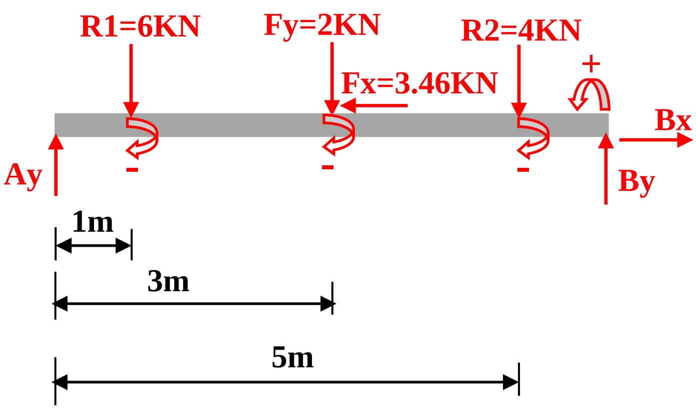
By × 6 – 4 ×5 – 2 × 3 – 6 ×1= 0

By × 6 – 20 – 6 – 6= 0

By × 6 = 32 ⇨ **By = 5.33KN**

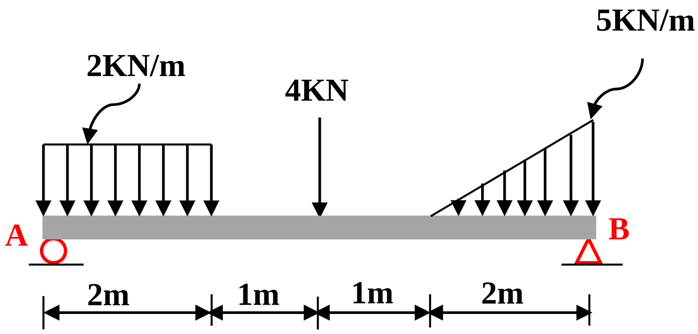
+↑ $\sum Fy = 0$

Ay + 5.33 – 6 -2 – 4 = 0 ⇨ **Ay = 6.67KN**



F.B.D of the beam

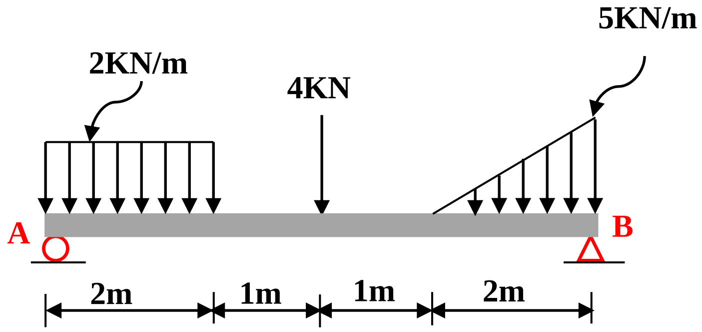
Example: Determine the reactions at supports (A) and (B)for the beam loaded as shown in figure .



Solution:

$+\uparrow$ R1 = - (2×2) = - 4KN = 4KN $\downarrow$

$+\uparrow$ R2 = - (1/2× 5×2) = - 5KN = 5KN $\downarrow$



$+ \rightarrow \sum F_x = 0 \quad \Rightarrow \quad B_x = 0$

$+ \circlearrowleft \sum M_A = 0$

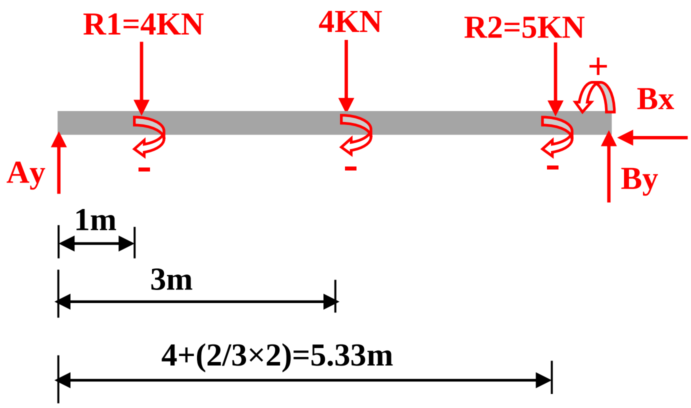
$B_y \times 6 - 5 \times 5.33 - 4 \times 3 - 4 \times 1 = 0$

$B_y \times 6 - 26.65 - 12 - 4 = 0$

$B_y \times 6 = 42.65 \Rightarrow B_y = 7.11KN$

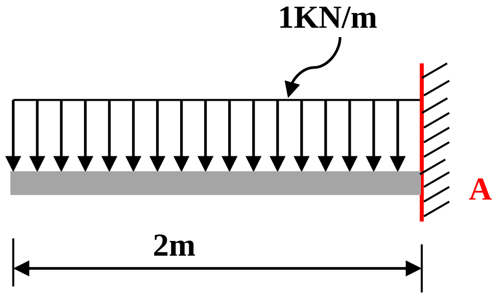
$+ \uparrow \sum F_y = 0$

$A_y + 7.11 - 4 - 4 - 5 = 0 \Rightarrow A_y = 5.89KN$

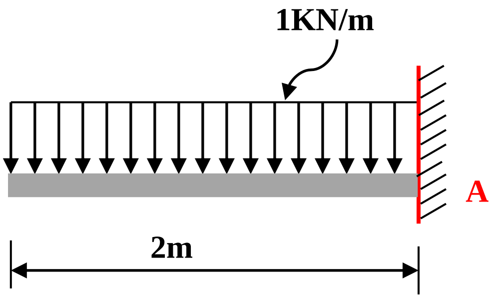


F.B.D of the beam

Example: Determine the reactions at support (A) for the beam loaded as shown in figure .



Solution:



$+\uparrow$ R $= -(2\times1) = -2KN = 2KN \downarrow$

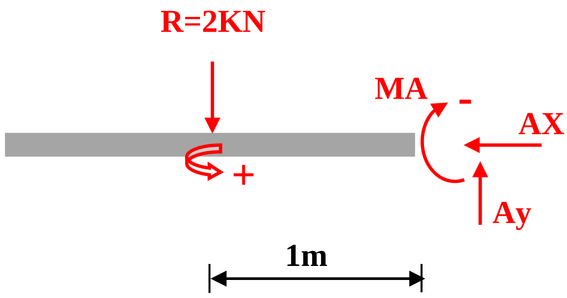
$+\rightarrow \sum Fx = 0 \Rightarrow Ax = 0$

$+\uparrow \sum Fy = 0$

$Ay - 2 = 0 \Rightarrow Ay = 2KN$

$+ \circlearrowleft \sum MA = 0$

$2\times 1 - MA = 0 \Rightarrow MA = 2KN.M$



TRUSSES:

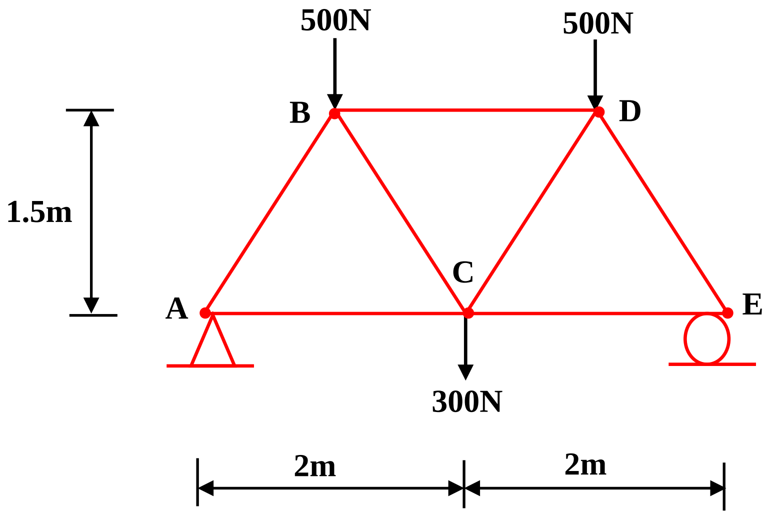
A truss is a structure composed of a number of members joined together at their ends to form a rigid body .

Analysis of trusses: is how to determine the forces in each member of the truss.

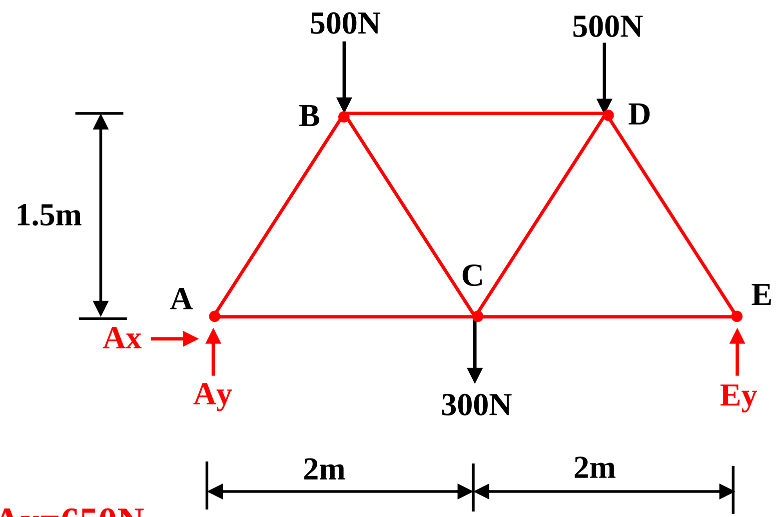
(1) : Method of joints : In this method a single joint is isolated as a free body diagram and applying the equations of concurrent forces.

$$\sum F_x = 0 \quad , \quad \sum F_y = 0$$

Example: Determine the forces in each member of the truss shown in figure and indicate if the member is in tension or compression.



Solution:



$+ \rightarrow \sum F_x = 0 \Rightarrow \quad A_x = 0$

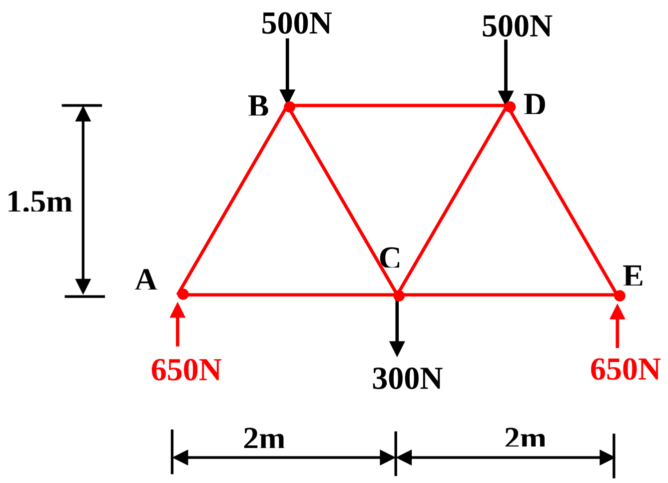
$+ \circlearrowleft \sum M_A = 0$

$$E_y \times 4 - 500 \times 1 - 500 \times 3 - 300 \times 2 = 0$$

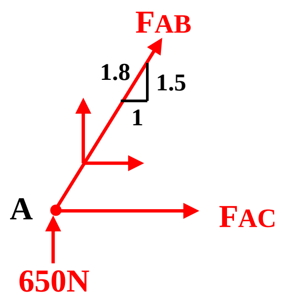
$E_y = 650N$

$+ \uparrow \sum F_y = 0$

$$A_y - 300 + 650 - 500 - 500 = 0 \quad \Rightarrow \quad A_y = 650N$$



Joint (A) :

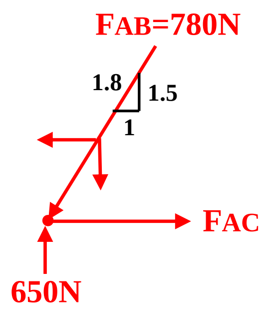


$+\uparrow \sum F_y = 0$

$650 + F_{AB} \times 1.5 / 1.8 = 0$

$F_{AB} = -780N = 780N$ (C)

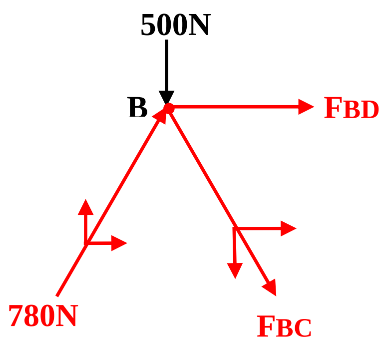
$+\rightarrow \sum F_x = 0$



$F_{AC} - 780 \times 1 / 1.8 = 0$

$F_{AC} = 433.3N$ (T)

Joint (B) :



$+\uparrow \sum F_y = 0$

$780 \times 1.5 / 1.8 - 500 - F_{BC} \times 1.5 / 1.8 = 0$

$F_{BC} = 180N$ (T)

$+\rightarrow \sum F_x = 0$

$780 \times 1 / 1.8 + F_{BD} + 180 \times 1 / 1.8 = 0$

$F_{BD} = -533.3N = 533.3N$ (C)

Joint (C) :

$+\uparrow \sum F_y=0$

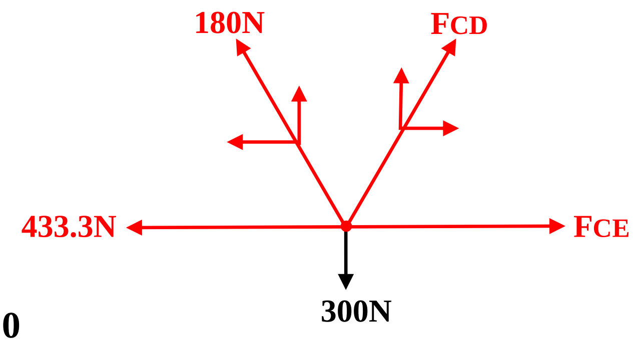
$180 \times 1.5 / 1.8 + F_{CD} \times 1.5 / 1.8 - 300 = 0$

$F_{CD}= 180N$ (T)

$+ \longrightarrow \sum F_x=0$

$F_{CE} - 180\times 1 / 1.8 + 180\times 1 / 1.8 - 433.3 = 0$

$F_{CE} = 433.3N$ (T)

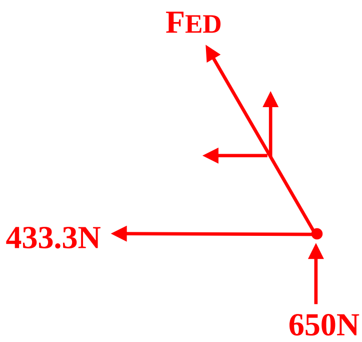


Joint (E) :

$+\uparrow \sum F_y=0$

$650 + F_{ED} \times 1.5 / 1.8 = 0$

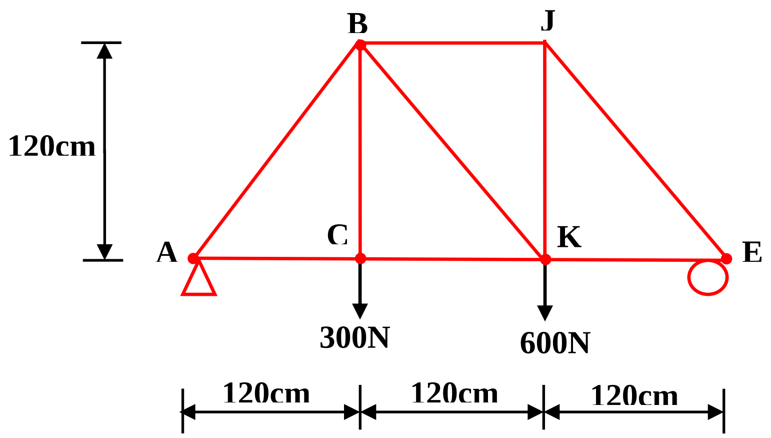
$F_{ED}= - 780N = 780N$ (C)



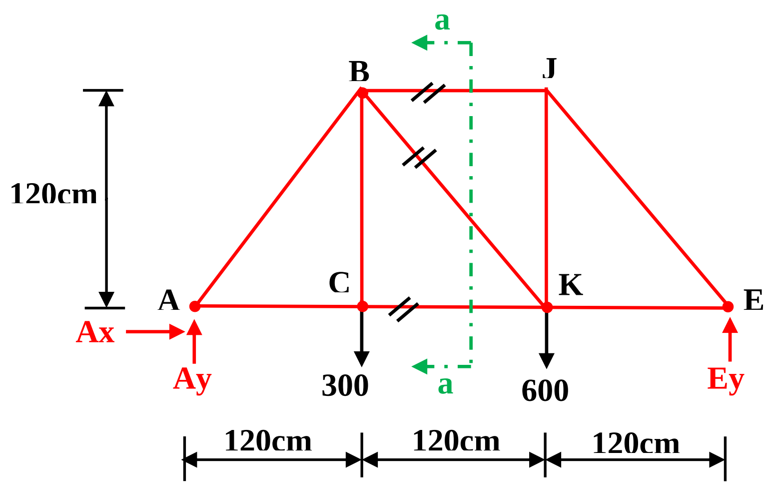
(2): Method Of Sections : When two or more joints are isolated and applying the equations of non-concurrent forces

$$\sum F_x=0 \quad , \quad \sum F_y=0 \quad , \quad \sum M=0.$$

Example: Determine the forces in members **(CK, BK, BJ)** for the truss shown in figure and indicate if the members are in tension or compression if **(Ay=400N)** and **(Ey=500N)**.



Solution:



From Section (a-a):

$+\uparrow \sum F_y = 0$

$400 - 300 - BK \times 1/\sqrt{2} = 0$

BK= 141.4N **(T)**

$+ \circlearrowleft \sum M_B = 0$

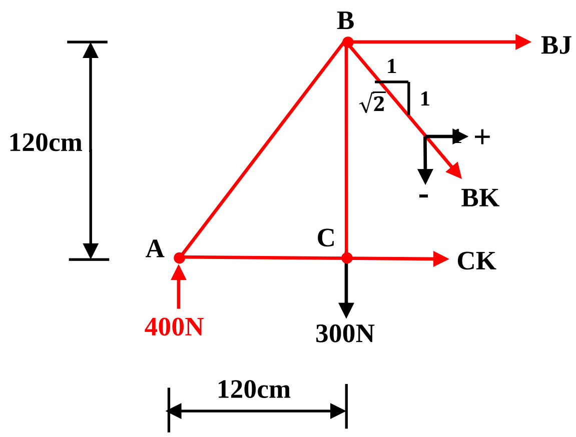
$CK \times 120 - 400 \times 120 = 0$

CK=400N **(T)**

$+ \rightarrow \sum F_x = 0$

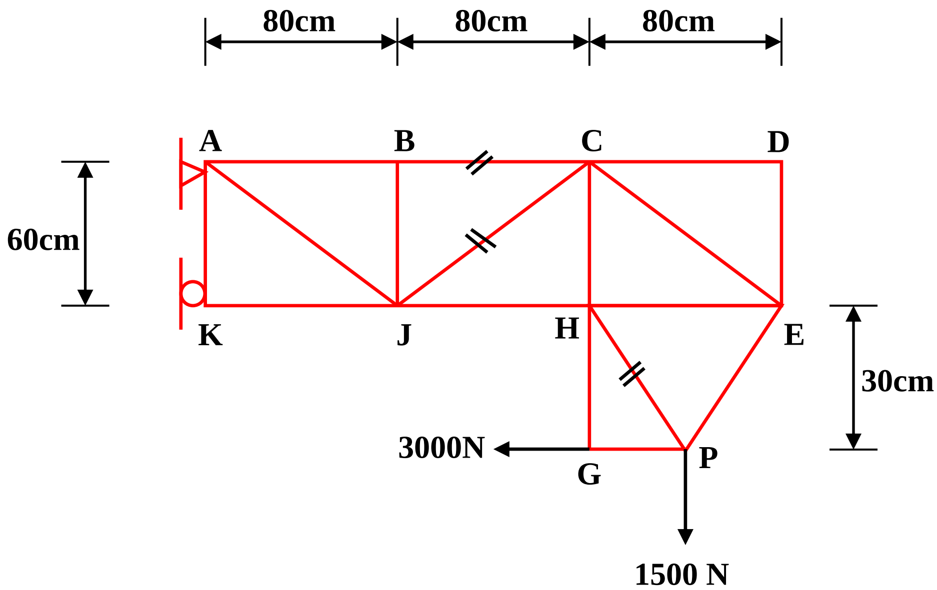
$400 + BJ + 141.4 \times 1/\sqrt{2} = 0$

BJ= -500N = 500N **(C)**



Sec.(a-a)

Example: Determine the forces in members (CJ, BC, HP) for the truss shown in figure and indicate if the members are in tension or compression .



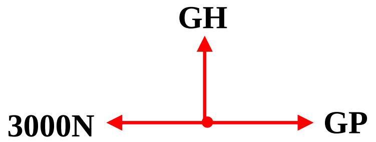
Solution:

Joint (G) :

$+\rightarrow \sum F_x=0$

GP- 3000 = 0

GP=3000N (T)



Joint (G)

Joint(P) :

$+\rightarrow \sum F_x=0$

$$PE*\frac{4}{5}-PH*\frac{4}{5}-3000=0 \quad \text{------------} \quad (1) \times 3$$

$+\uparrow \sum F_y=0$

$$PE*\frac{3}{5}+PH*\frac{3}{5}-1500=0 \quad \text{------------} \quad (2) \times 4$$

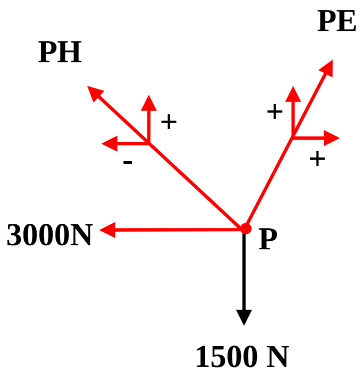
$$PE*\frac{12}{5}-PH*\frac{12}{5}-9000=0 \quad \text{------------} \quad (1)$$

$$PE*\frac{12}{5}+PH*\frac{12}{5}-6000=0 \quad \text{------------} \quad (2)$$

+

$$PE*\frac{24}{5}=15000$$

$$PE=3125N(T)$$

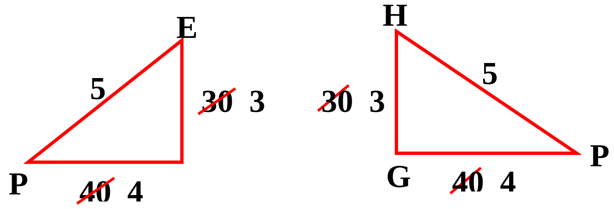


Joint (P)

Sub. In equation (1)

$$3125*\frac{4}{5}-PH*\frac{4}{5}-3000=0$$

$$PH=-625N=625N(C)$$



From section (1-1) :

$+\uparrow\sum Fy=0$

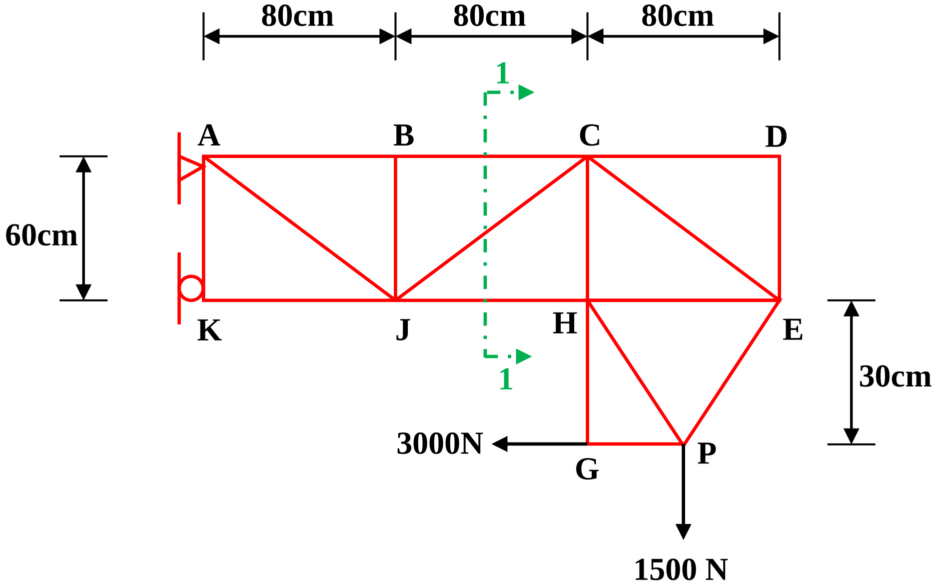
$$-CJ*\frac{3}{5}-1500=0$$

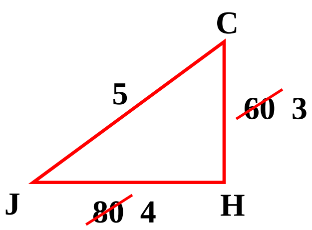
$$CJ=-2500N=2500N(C)$$

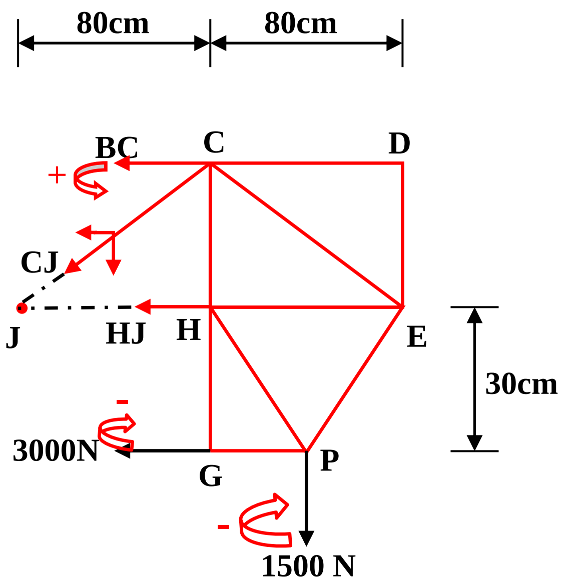
$+\circlearrowleft\sum MJ=0$

$$-1500*120-3000*30+BC*60=0$$

$$BC=4500N(T)$$







FRICTION: Is the force tangent to the contact surface which resists the motion when a body slides or tends to slides on another body .

Friction Theory: Let a block of weight (W) rests on a horizontal plane as shown in (Figure 1), and a horizontal force (P) is applied on it as shown in (Figure 2):

1:-When (P=0) the frictional force (F=0) and the block is in equilibrium.

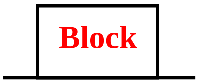


Figure 1

2:-When (P) increased the frictional force (F) is also increased in the same value to prevent motion.

3:-When (F) reach its maximum value (Fmax.) any increase in (P) will cause motion.

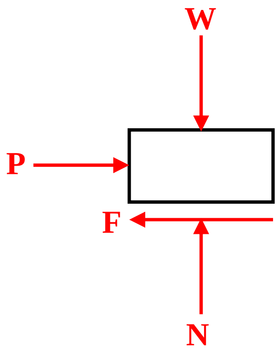


Figure 2

Laws of friction: The maximum frictional force (Fmax.) is proportional with the normal force (N) between the contact surfaces .

$$F_{max.} \propto N$$

$$\boxed{F_{max.} = \mu \times N} \implies \mu = F_{max.} / N$$

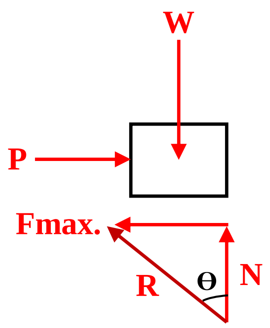
μ: coefficient of friction and depends on the roughness of surfaces

Angle of friction:

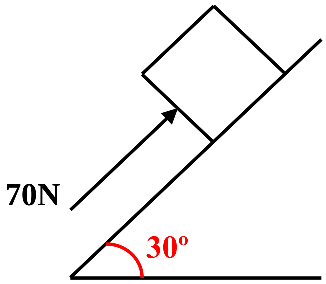
$$\tan \Theta = F_{max.} / N$$

$$\mu = F_{max.} / N$$

$$\boxed{\tan \Theta = \mu}$$



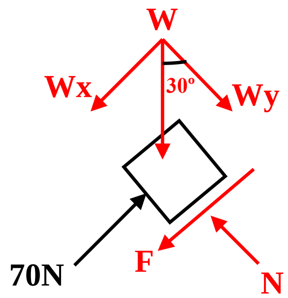
Example: Determine the frictional force exerted on the (200N) block weight by the inclined surface shown in figure if the block is subjected to (70N) force, ($\mu=0.2$).



Solution:

$Wx = 200 \times \sin 30 = 100N$

$Wy = 200 \times \cos 30 = 173.2N$



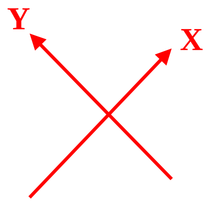
Assume the block will move upward

$+ \nearrow \sum Fx = 0$

$70 - 100 - F = 0$

$F = -30N$

That means the block is try to move downward
(F) must be equal or less than (Fmax.)



$Fmax. = \mu \times N$

$+ \nwarrow \sum Fy = 0$

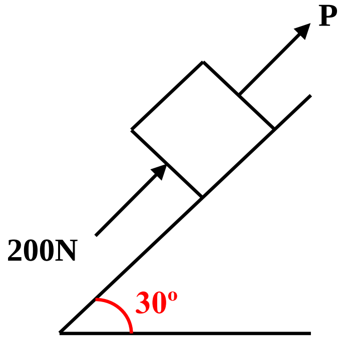
$N - 173.2 = 0$

$N = 173.2N$

$Fmax. = 0.2 \times 173.2 = 34.64N > 30N$

$F = 30N$

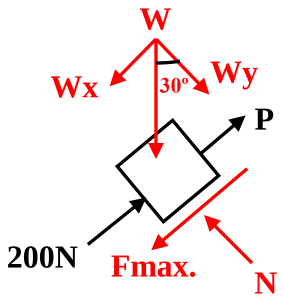
Example: Calculate the force (P) required to move the (500N) block weight up the inclined surface shown in figure, if the block is subjected to (200N) force, assume (μ=0.5) .



Solution:

$W_x = 500\times \sin 30 = 250N$

$W_y = 500\times \cos 30 = 433N$



+↖ $\sum F_y = 0$

N - 433 = 0

N = 433N

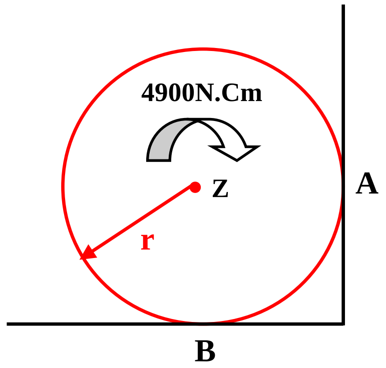
$F_{max.} = \mu\times N = 0.5\times 433 = 216.5N$

+↗ $\sum F_x = 0$

200 + P - 250 - 216.5 = 0

P = 266.5N

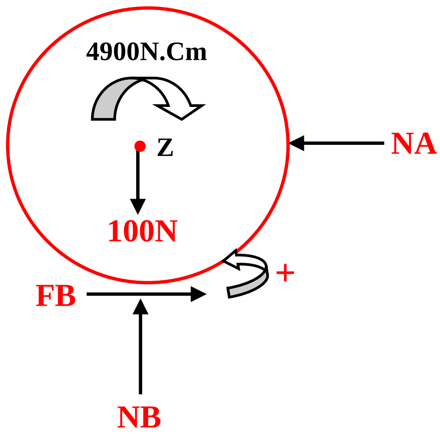
Example: A cylinder of (100N) weight, (r=70cm) is to entrust to a horizontal surface its coefficient of friction (μ=0.4) and a smooth vertical surface as shown in figure .Determine the frictional force .



Solution:

From F.B.D of cylinder

Assume FB to the right as shown



(F.B.D of cylinder)

$+\circlearrowleft \sum M_Z=0$

$-4900 + FB \times 70 = 0$

$FB = 70N$

FB must be equal or less than (Fmax.)

$Fmax. = \mu \times N$

$+\uparrow \sum Fy = 0$

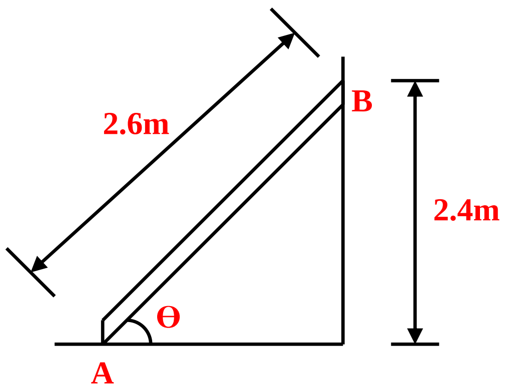
$NB - 100 = 0$

$NB = 100N$

$Fmax. = 0.4 \times 100 = 40N < 70N$

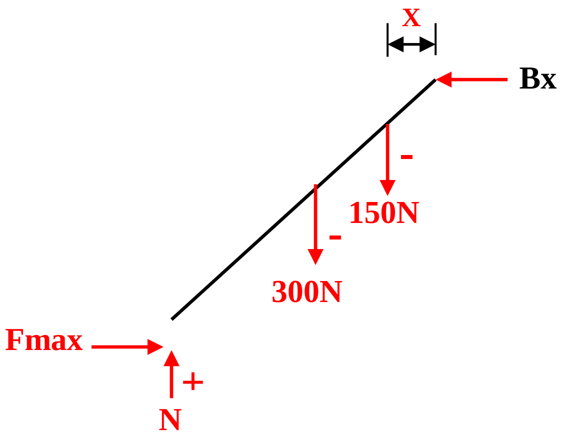
$FB = 40N$

Example: A ladder **(300N)** weight is rest as shown in figure, if the vertical wall is smooth and the horizontal surface has **(μ=0.2)**. Determine the distance from point **(B)** which make the ladder move when a boy of **(150N)** weight try to going up the ladder .



Solution:

From F.B.D of ladder:



(F.B.D of ladder)

$+\uparrow \sum F_y = 0$

$N - 300 - 150 = 0$

$N = 450N$

$F_{max.} = \mu \times N$

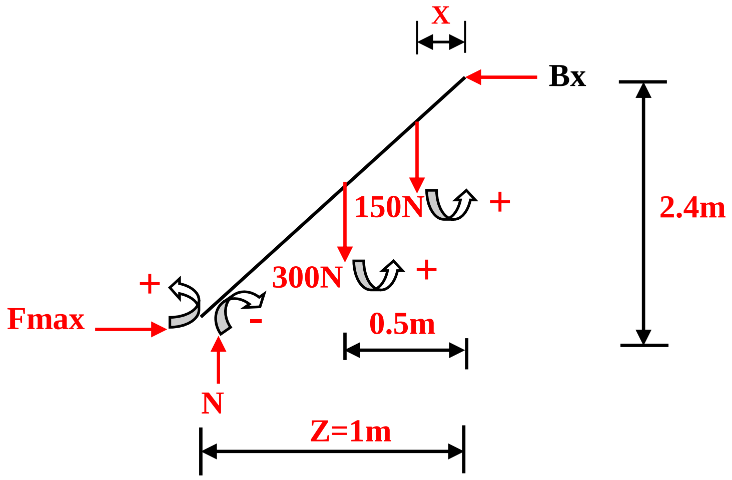
$= 0.2 \times 450 = 90N$

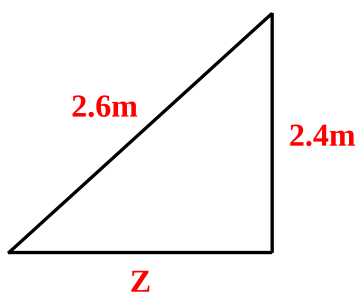
$Z = \sqrt{(2.6)^2 - (2.4)^2} = 1m$

$+\circlearrowright \sum M_B = 0$

$-450 \times 1 + 300 \times 0.5 + 90 \times 2.4 + 150 \times X = 0$

$X = 0.56m$



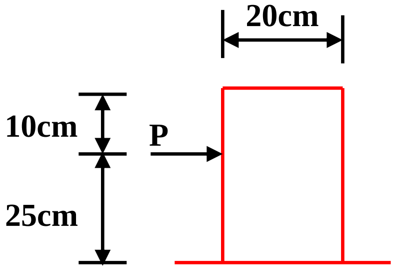


$(2.4)^2 + (Z)^2 = (2.6)^2$

$Z^2 = (2.6)^2 - (2.4)^2$

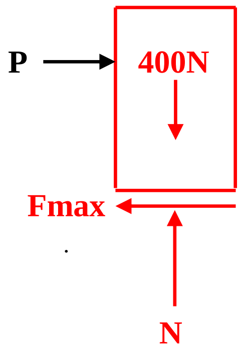
$Z = \sqrt{(2.6)^2 - (2.4)^2} = 1m$

Example: Determine the force **(P)** required to move the **(400N)** block weight shown in figure if the horizontal surface has **(μ =0.34)** .



Solution:

The block is either slides or overturn



F.B.D 1

1-the block is slides From (F.B.D 1)

$+\uparrow \sum F_y = 0$

$N - 400 = 0$

$N = 400N$

$F_{max.} = \mu \times N$

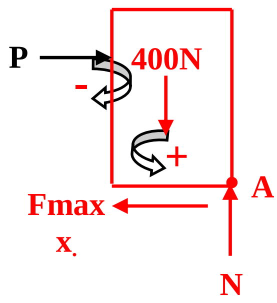
$= 0.34 \times 400 = 136N$

$+ \longrightarrow \sum F_x = 0$

$P - 136 = 0$

$P = 136$

2-the block is overturn From (F.B.D 2)



F.B.D 2

$+ \circlearrowleft \sum M_A = 0$

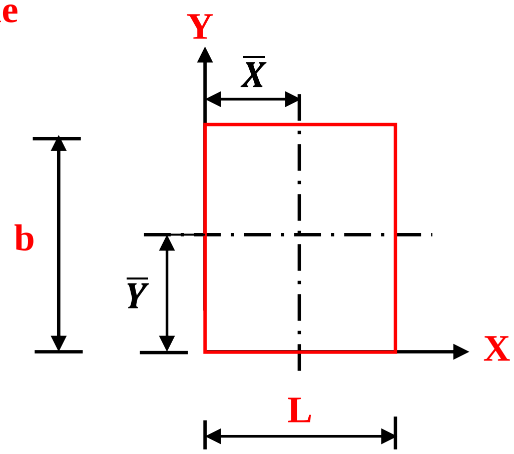
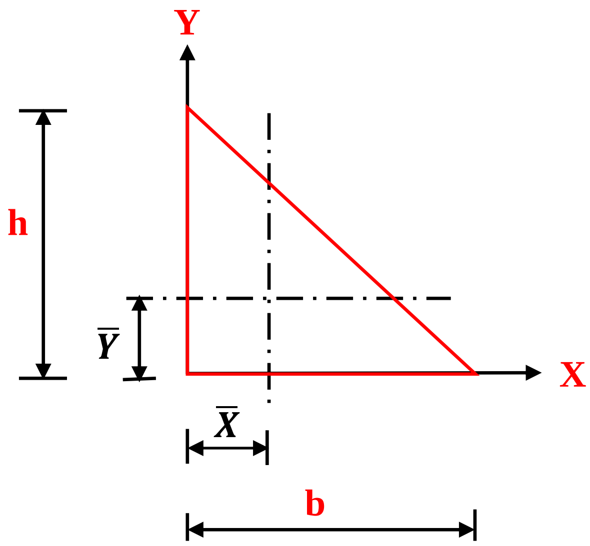
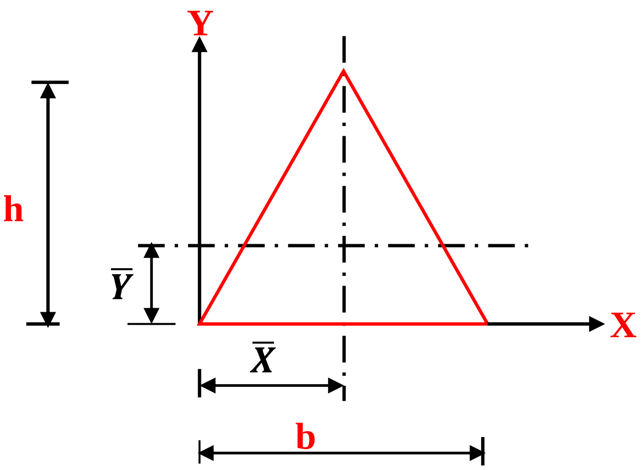
$- P \times 25 + 400 \times 10 = 0$

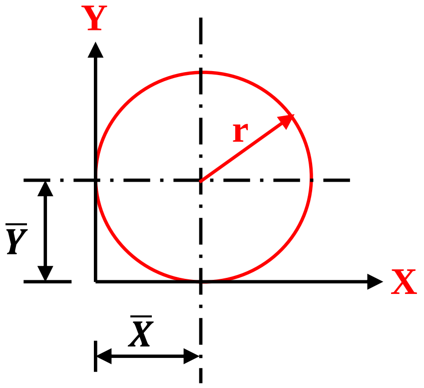
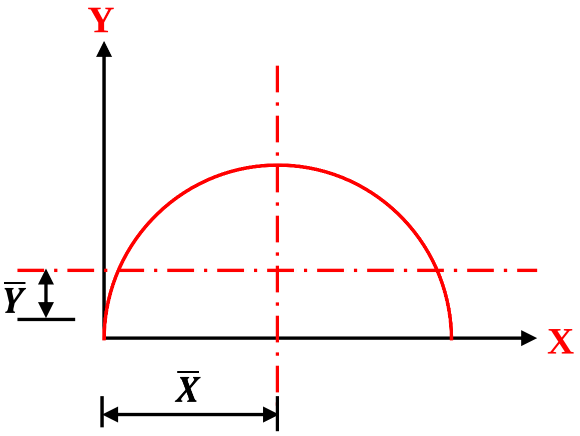
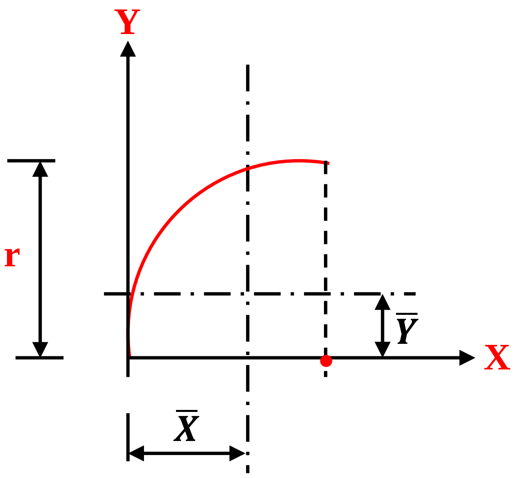
$P = 160N$

The block is slides and P = 136N

CENTROID

1: Centroids of simple shapes

Shape	Area (a_i)	$\bar{X}$	$\bar{Y}$
1-Rectangle 	L × b	L / 2	b / 2
2-Triangle 	1/2× b×h	b / 3	h / 3
	1/2× b×h	b / 2	h / 3

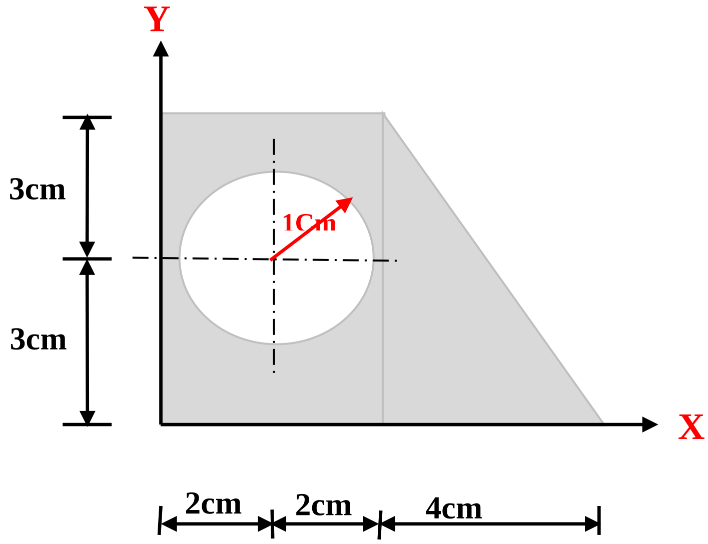
<table>
<tr><th></th><th></th><th></th><th></th></tr>
<tr><td>3- Circle

</td><td>πr^2</td><td>r</td><td>r</td></tr>
<tr><td>4-Half circle

</td><td>$\pi r^2 / 2$</td><td>r</td><td>$0.424 \times r$</td></tr>
<tr><td>5-Quarter circle

</td><td>$\pi r^2 / 4$</td><td>$r - 0.424 \times r$</td><td>$0.424 \times r$</td></tr>
</table>

2 : Centroids of complex shapes:

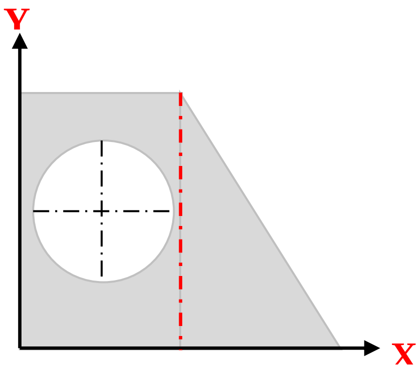
NOTE: **the coordinates $(\bar{X},\bar{Y})$ of centroid of any non-uniformly area about (X) and (Y) axes can be found by :**

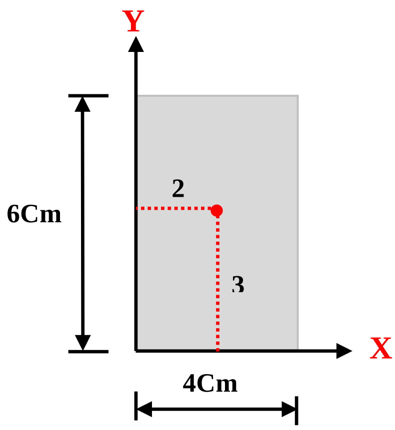
$$\bar{X}=\frac{\sum a_i x_i}{\sum a_i} \quad , \quad \bar{Y}=\frac{\sum a_i y_i}{\sum a_i}$$

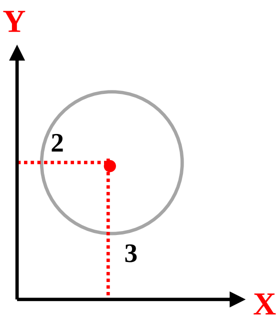
Example: Determine the centroid of the shaded area shown in figure with respect to (X) and (Y) axes .



Solution:







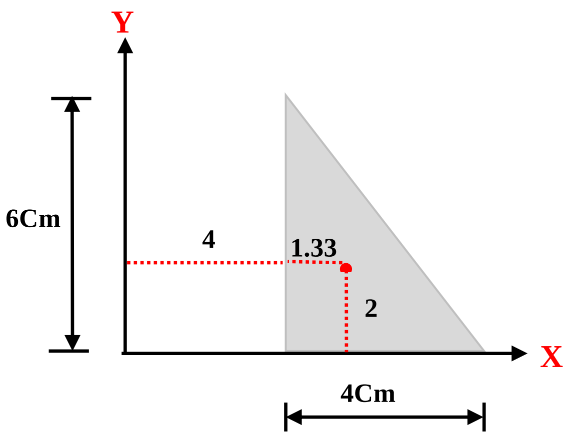
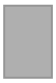
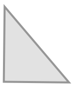
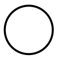
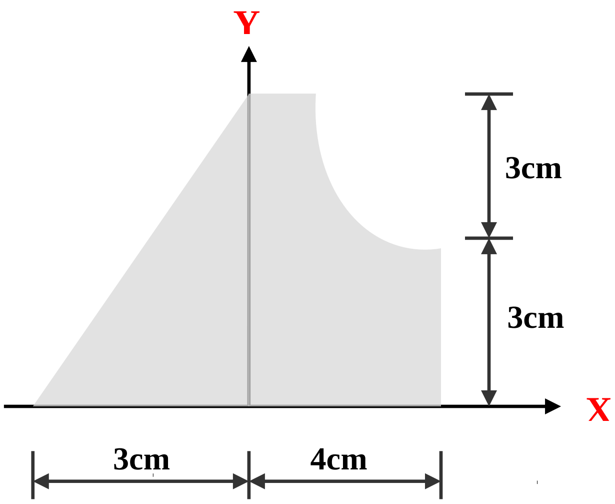


Fig.	a_i	x_i	y_i	$a_i\ x_i$	$a_i\ y_i$
	**4 × 6 = 24**	**2**	**3**	**48**	**72**
	**1/2 × 4 × 6 = 12**	**4 + 1.33 = 5.33**	**2**	**64**	**24**
	**- π (1)²=-3.14**	**2**	**3**	**- 6.28**	**- 9.42**
$\sum$	**32.86**			**105.72**	**86.58**

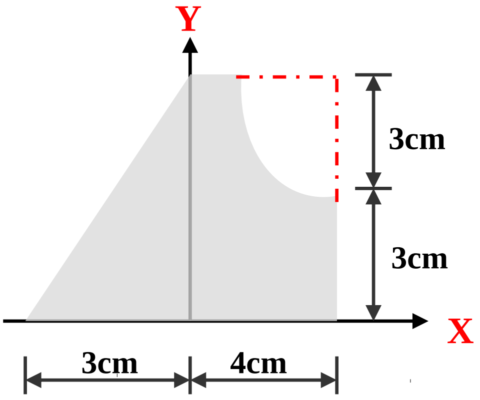
$$\bar{X} = \frac{\sum a_i x_i}{\sum a_i}$$ **= 105.72 / 32.86 =3.2 Cm** ,

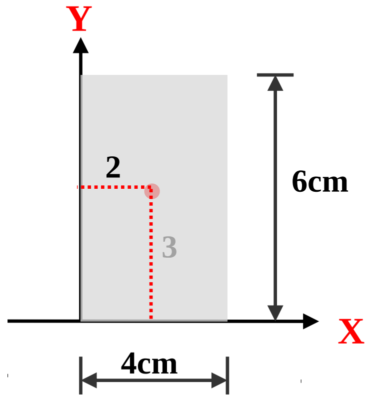
$$\bar{Y} = \frac{\sum a_i y_i}{\sum a_i}$$ **= 86.58 / 32.86 = 2.6 Cm**

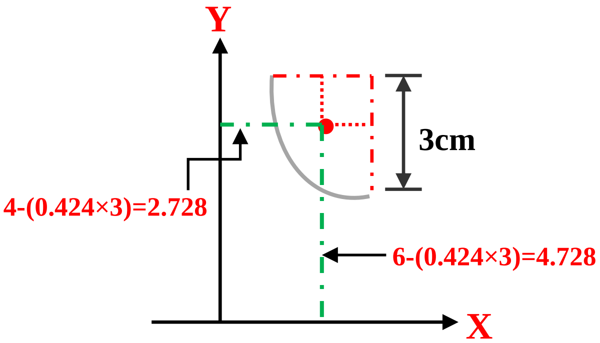
Example: Determine the centroid of the shaded area shown in figure with respect to (X) and (Y) axes.



Solution:







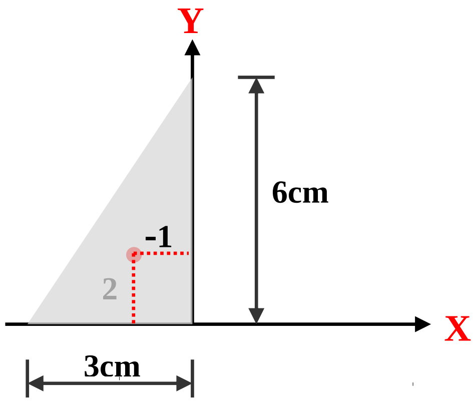
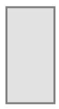
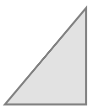
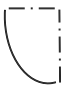
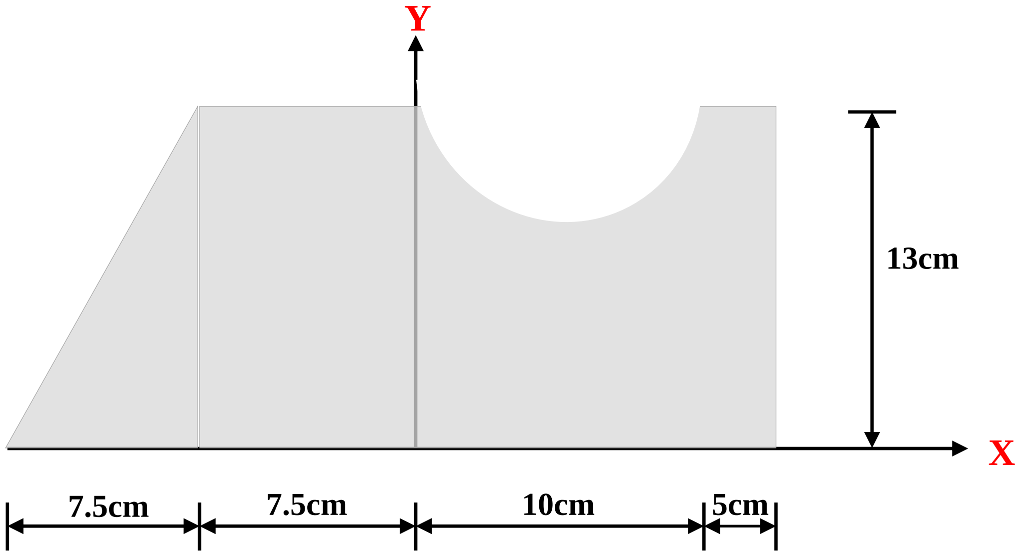


Fig.	a_i	x_i	y_i	$a_i\ x_i$	$a_i\ y_i$
	4 × 6 =24	2	3	48	72
	1/2 × 3 × 6 = 9	- 1	2	- 9	18
	$-\pi(3)^2/4 = -7.069$	4 - (0.424 ×3) =2.728	6 - (0.424 × 3) = 4.728	- 19.27	-33.4
$\sum$	25.931			19.73	56.6

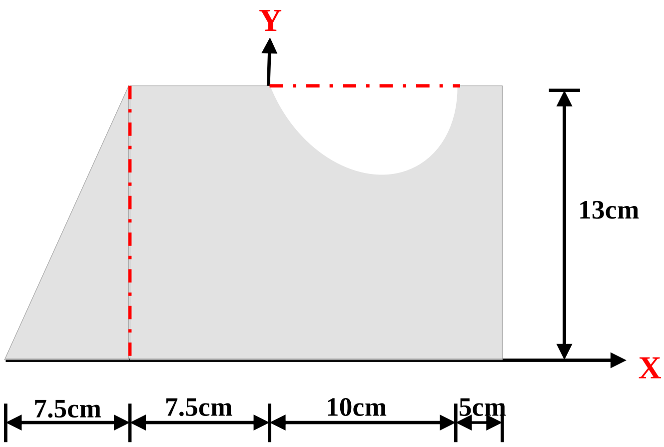
$\bar{X} = \frac{\sum a_i x_i}{\sum a_i}$ =19.73/25.931= 0.76cm

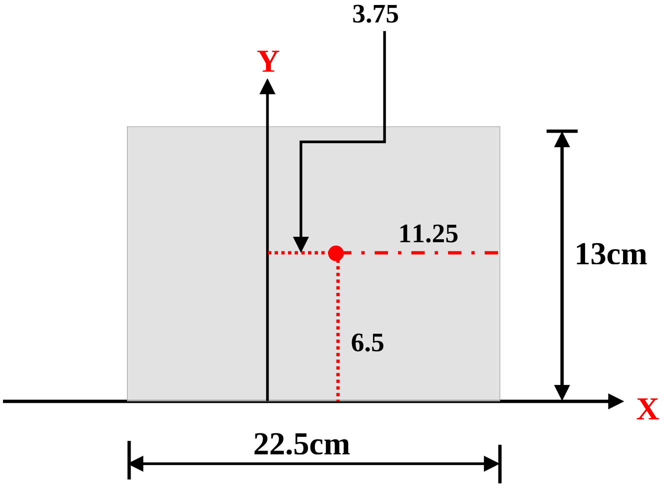
$\bar{Y} = \frac{\sum a_i y_i}{\sum a_i}$ = 56.6/25.931=2.18cm

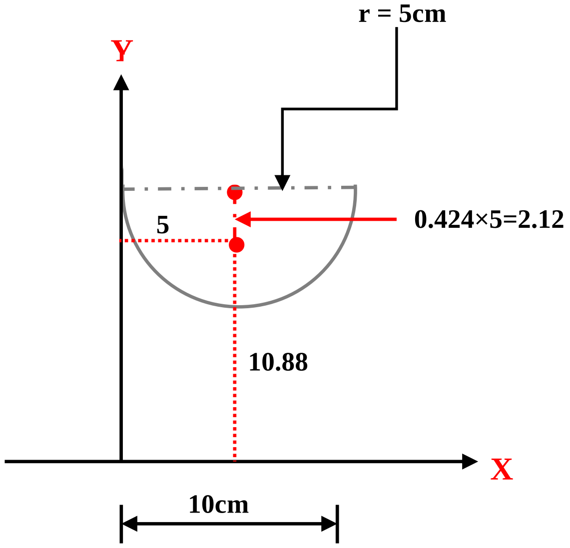
Example: Determine the centroid of the shaded area shown in figure with respect to (X) and (Y) axes.



Solution:







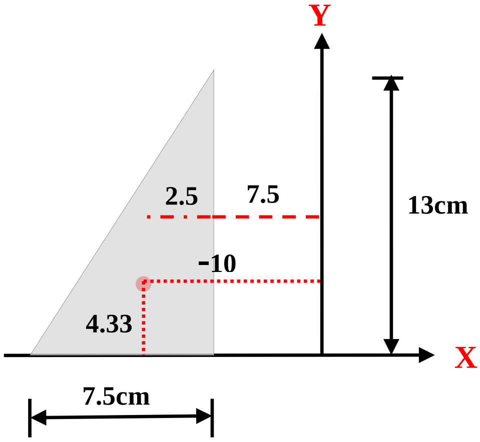
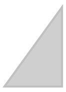
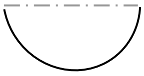
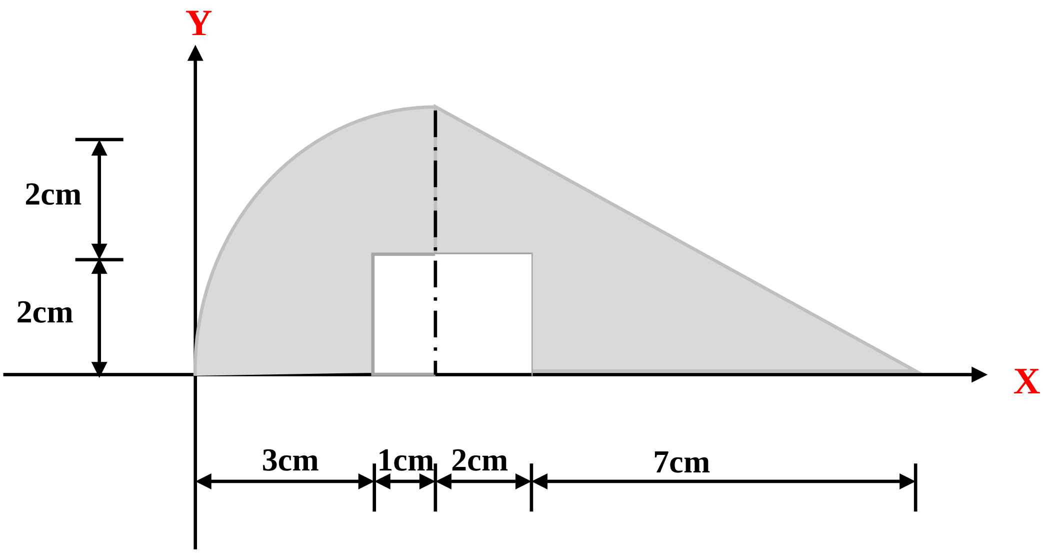


Fig.	a_i	x_i	y_i	$a_i\ x_i$	$a_i\ y_i$
	**22.5×13 = 292.5**	**15-11.25=3.75**	**6.5**	**1096.875**	**1901.25**
	**1/2× 7.5×13 = 48.75**	**- 10**	**4.33**	**- 487.5**	**211.25**
	**- π(5)²/2 = - 39.26**	**5**	**10.88**	**- 196.3**	**- 427.14**
$\sum$	**301.99**			**413.075**	**1685.36**

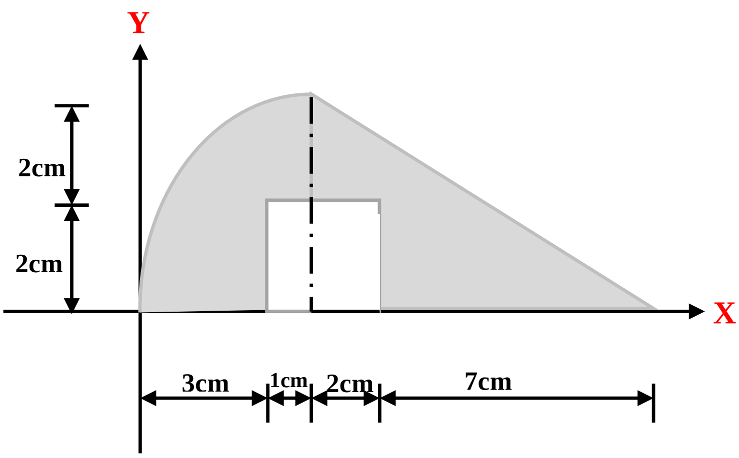
$\bar{X} = \frac{\sum a_i x_i}{\sum a_i}$ **=413.075/301.99 = 1.36cm**

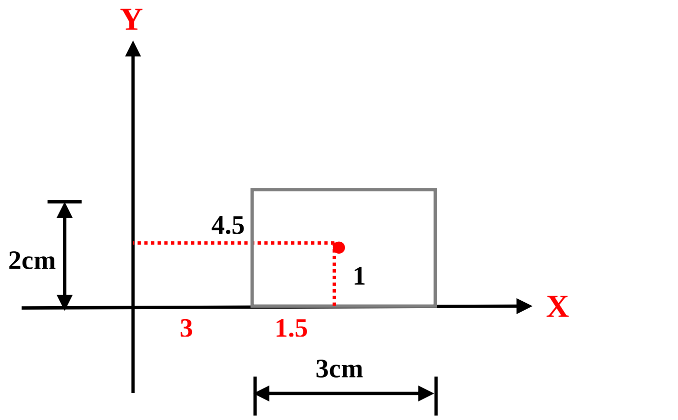
$\bar{Y} = \frac{\sum a_i y_i}{\sum a_i}$ **= 1685.36/301.99 = 5.58cm**

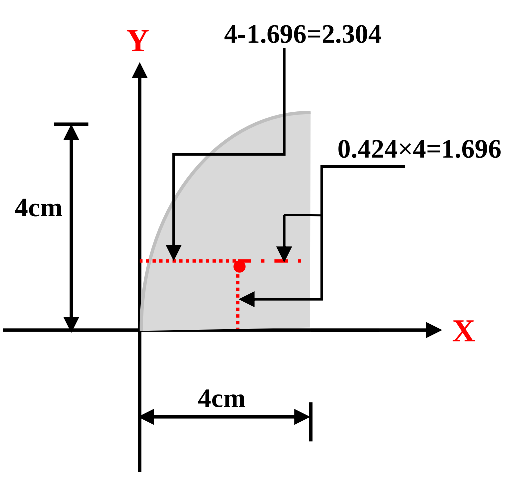
Example: Determine the centroid of the shaded area shown in figure with respect to (X) and (Y) axes.



Solution:







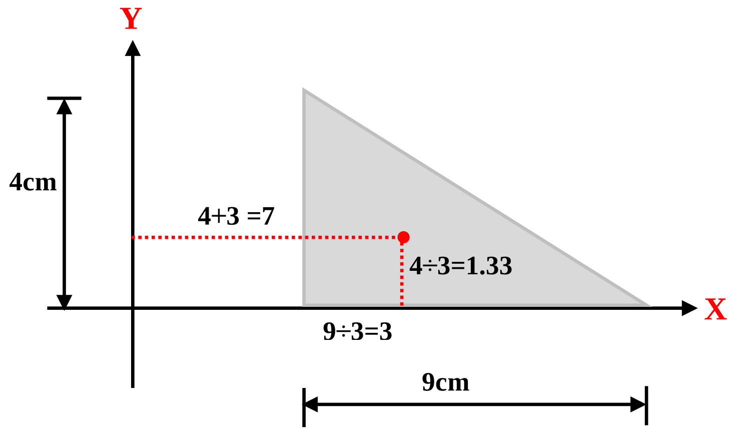
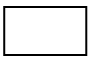
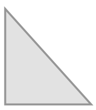
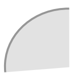


Fig.	a_i	x_i	y_i	$a_i\ x_i$	$a_i\ y_i$
	- (3× 2)=-6	4.5	1	- 27	- 6
	1/2× 9× 4 =18	7	1.33	126	23.94
	$\pi(4)^2/4$ =12.56	4- 1.696=2.304	1.696	28.93	21.3
$\sum$	24.56			127.93	39.24

$\bar{X} = \frac{\sum a_i x_i}{\sum a_i}$ =127.93/24.56=5.2cm

$\bar{Y} = \frac{\sum a_i y_i}{\sum a_i}$ = 39.24/24.56=1.59cm

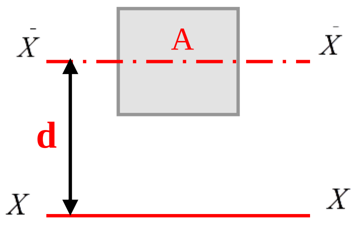
Moment of Inertia: **(I)**

The moment of inertia of an area is equal to the product of this area by the square distance about the axis of rotation.

$$I = A \times d^2$$

Transfer formula for moment of inertia :

$$I_x = I\bar{x} + A \times d^2$$



Units of moment of inertia: mm^4 **,** Cm^4

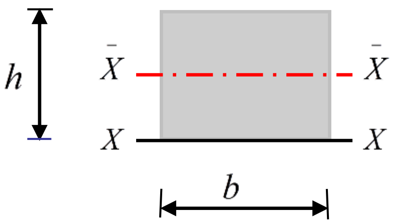
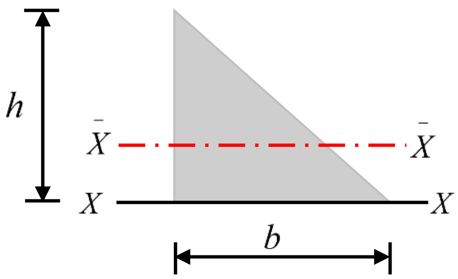
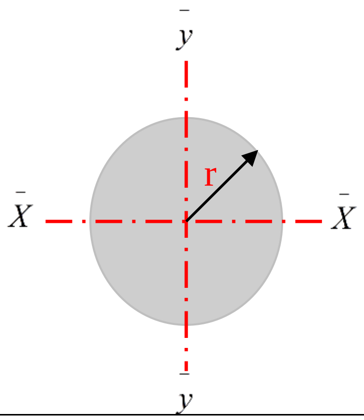
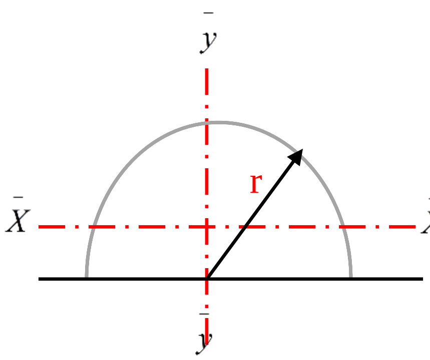
Polar moment of inertia: **Ij_0**

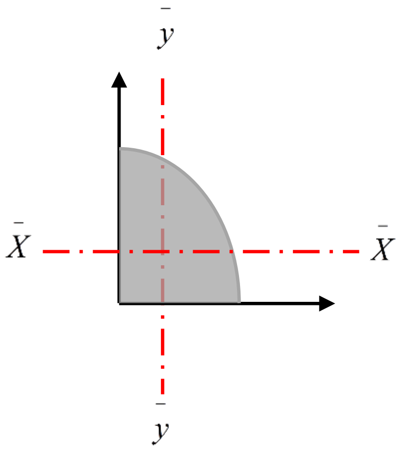
Ij_o=Ix+Iy

Radius of gyration: **K_x**

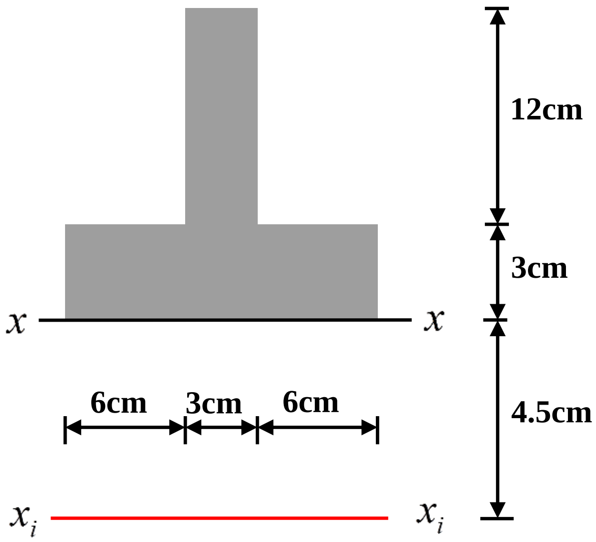
K_x= $\sqrt{I/A}$

① :-Moment of inertia for the simple shapes:

Shape	Moment of Inertia (I)	Radius of gyration (K)
	$I_{\bar{x}} = \frac{bh^3}{12}$ $I_x = \frac{bh^3}{3}$	$K_{\bar{x}} = \frac{h}{\sqrt{12}}$ $K_x = \frac{h}{\sqrt{3}}$
	$I_{\bar{x}} = \frac{bh^3}{36}$ $I_x = \frac{bh^3}{12}$	$K_{\bar{x}} = \frac{h}{\sqrt{18}}$ $K_x = \frac{h}{\sqrt{6}}$
	$I_{\bar{X}} = I_{\bar{y}} = \frac{\pi r^4}{4}$	
	$I_{\bar{X}} = 0.11r^4$ $I_X = I_{\bar{y}} = \frac{\pi r^4}{8}$	$K_x = K_{\bar{y}} = \frac{r}{2}$ $K_{\bar{x}} = 0.264r$

	$I_x = I_y = \dfrac{\pi r^4}{16}$ $I_{\bar{x}} = I_{\bar{y}} = 0.055 r^4$	$K_x = K_y = \dfrac{r}{2}$ $K_{\bar{x}} = K_{\bar{y}} = 0.264 r$

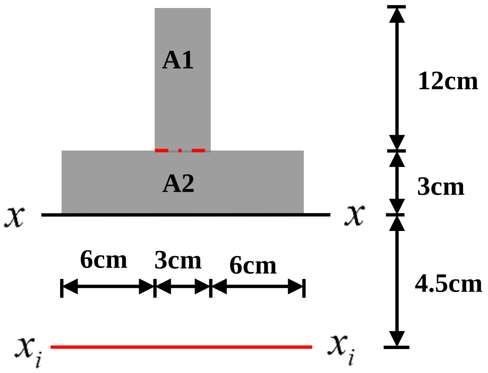
Example: Determine the moment of inertia of the shaded area shown in figure with respect to ($x_i - x_i$) axis .



Solution:

A1=12× 3 =36cm²

A2 =15× 3 =45cm²

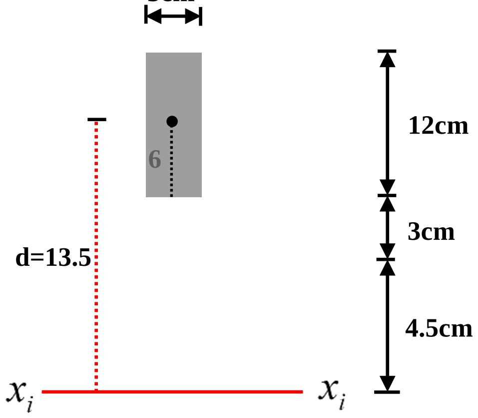


For (A1):

$$I_{x_i} = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2$$

= 3 × (12)³/12+36 × (13.5)² = 6993 cm^4 (+)



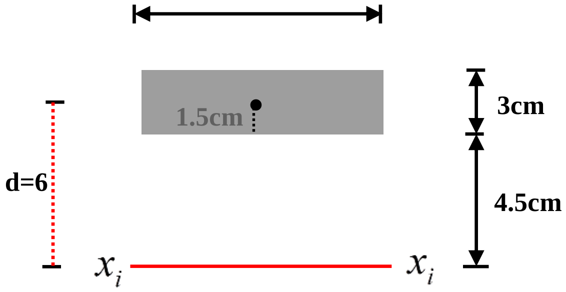
A1

For (A2):

$$I_{x_i} = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2$$

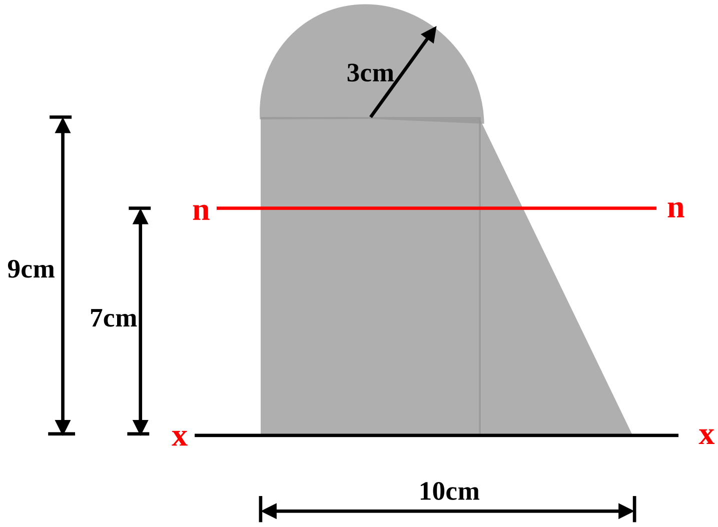
=15 × (3)³/12+45 × (6)² =1653.75 cm^4 (+)



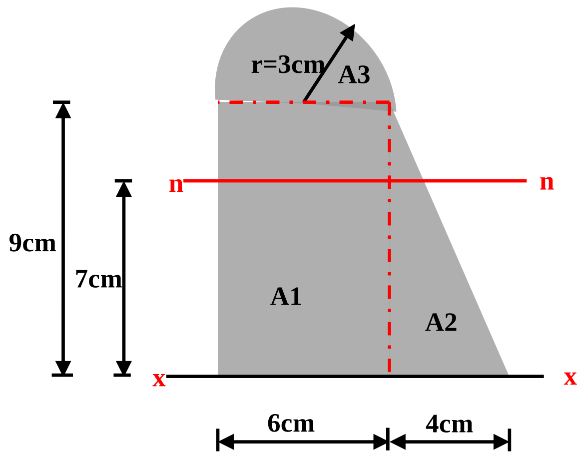
A2

I_{x_i} (total) = 6993+ 1653.75 = 8646.75 cm^4

Example: **Determine the moment of inertia of the shaded area shown in figure with respect to (n-n) axis.**



Solution:



A1 = 6 × 9 = 54 Cm²

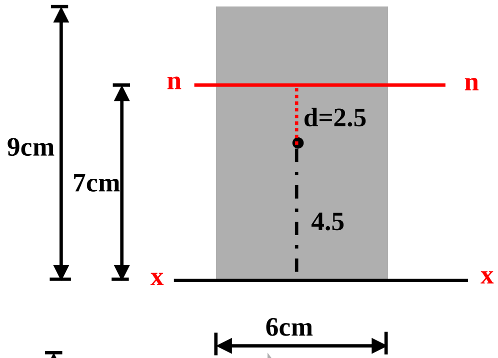
A2 = 1/2 × 4 × 9 = 18 Cm²

A3 = π × (3)²/2 = 14.14 Cm²

For (A_1):

$$In = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2 = 6 \times (9)^3/12 + 54 \times (2.5)^2 = 702\,cm^4 \quad (+)$$

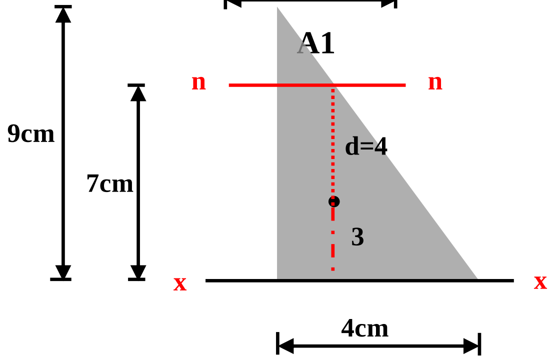


A1

For (A_2):

$$In = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{36} + Ad^2 = 4 \times (9)^3/36 + 18 \times (4)^2 = 369\,cm^4 \quad (+)$$



A2

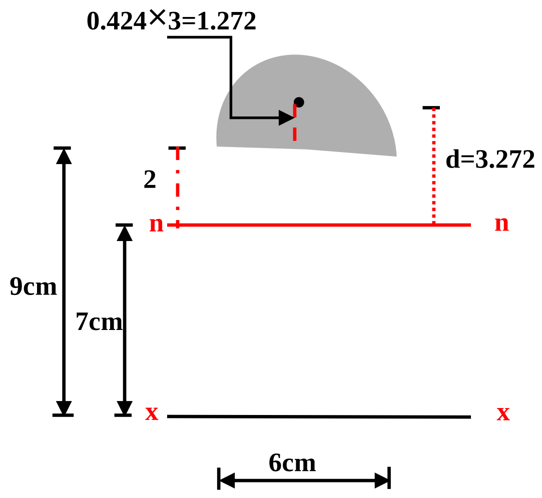
For (A_3):

$I_n = I_{\bar{x}} + Ad^2$

$= 0.11\, r^4 + Ad^2 = 0.11 \times (3)^4 + 14.14 \times (3.272)^2$

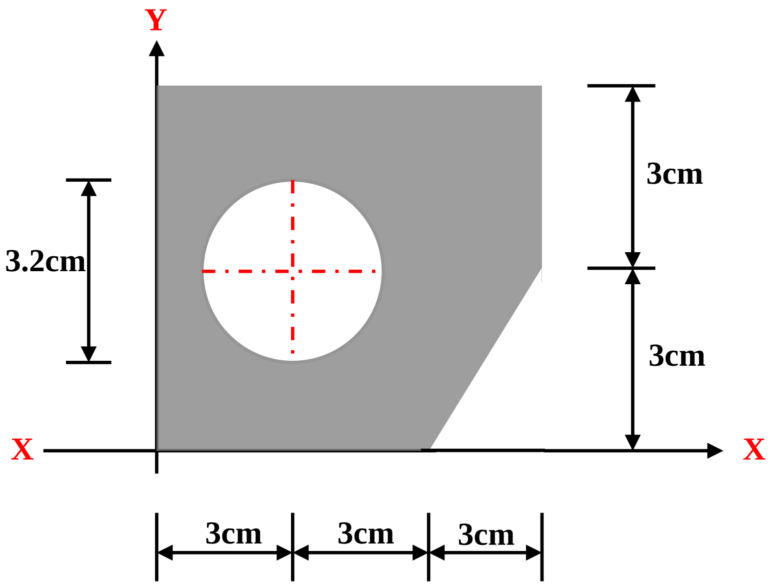
$= 160.29\, cm^4$ (+)

$I_n(total) = 702 + 369 + 160.29 = 1231.29\, cm^4$

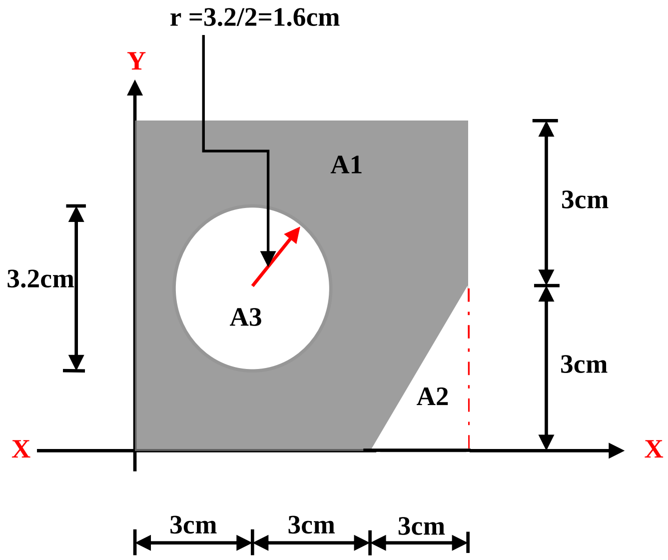


A3

Example: **Determine the moment of inertia of the shaded area shown in figure with respect to (x-x) axis .**



Solution:



$A1 = 6 \times 9 = 54 cm^2$

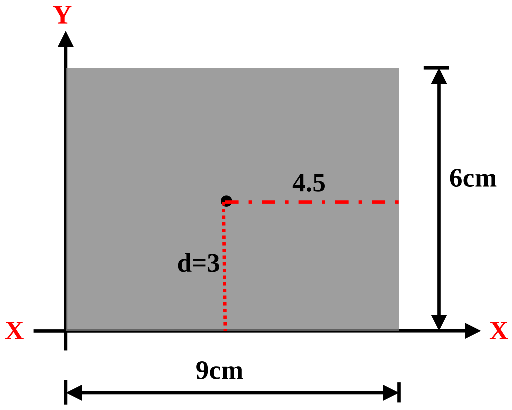
$A2 = 1/2 \times 3 \times 3 = 4.5 cm^2$

$A3 = \pi \times (1.6)^2 = 8.04 cm^2$

For (A_1):

$$I_x = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2 = 9\times(6)^3/12 + 54\times(3)^2 = 648\ cm^4 \quad (+)$$

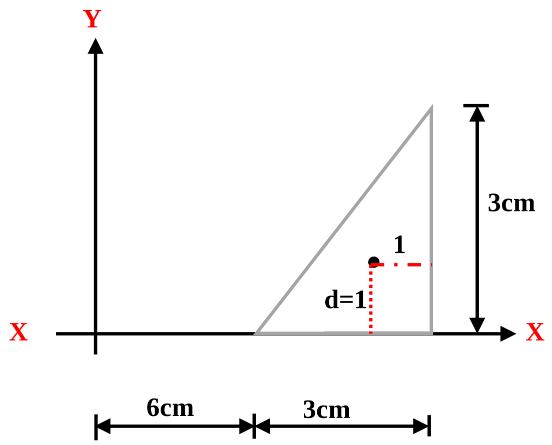


A1

For (A_2):

$$I_x = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{36} + Ad^2 = 3\times(3)^3/36 + 4.5\times(1)^2 = 6.75\ cm^4 \quad (-)$$

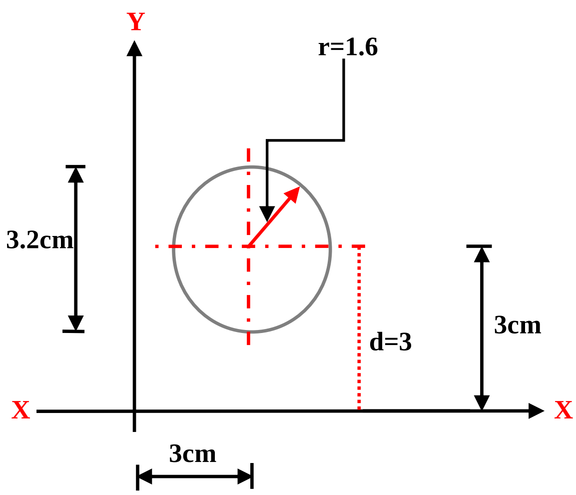


A2

For (A_3):

$$I_x = I_{\bar{x}} + Ad^2$$

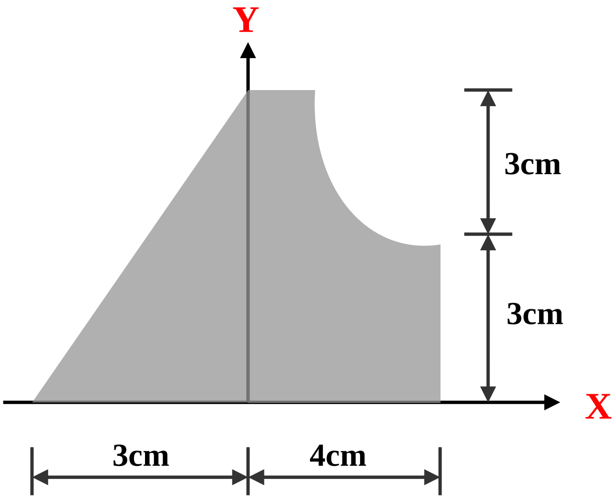
$$= \frac{\pi r^4}{4} + Ad^2 = \pi\times(1.6)^4/4 + 8.04\times(3)^2 = 77.7\ cm^4 \quad (-)$$



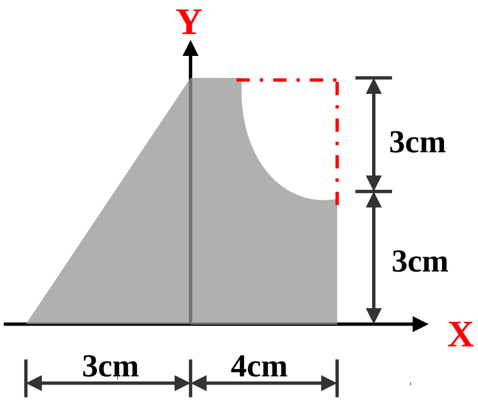
A3

I_x (total) = 648 - 6.75 - 77.5 = 563.75 cm^4

Example: Determine the moment of inertia of the shaded area shown in figure with respect to (x) axis .



Solution:



$A_1 = 4 \times 6 = 24 Cm^2$

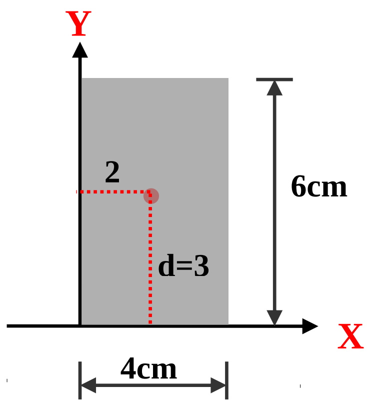
$A_2 = 1/2 \times 3 \times 6 = 9 Cm^2$

$A_3 = \pi (3)^2 / 4 = 7.06 Cm^2$

For (A_1) :

$$Ix = I_{\bar{x}} + Ad^2$$

$$I_x = \frac{bh^3}{12} + Ad^2 = 4 \times (6)^3/12 + 24 \times (3)^2 = 288 Cm^4 \quad (+)$$

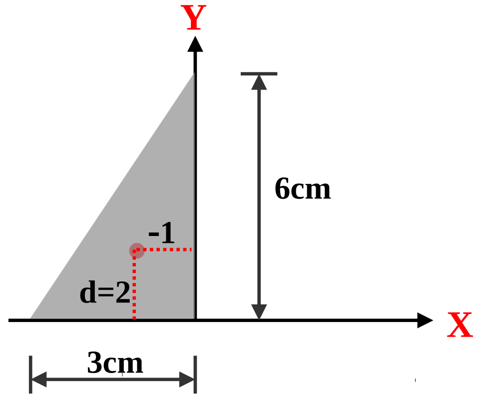


A1

For (A_2) :

$Ix = I_{\bar{x}} + Ad^2$

$I_x = \frac{bh^3}{36} + Ad^2$ **$=3\times (6)^3/36 + 9\times (2)^2 = 54Cm^4$ (+)**

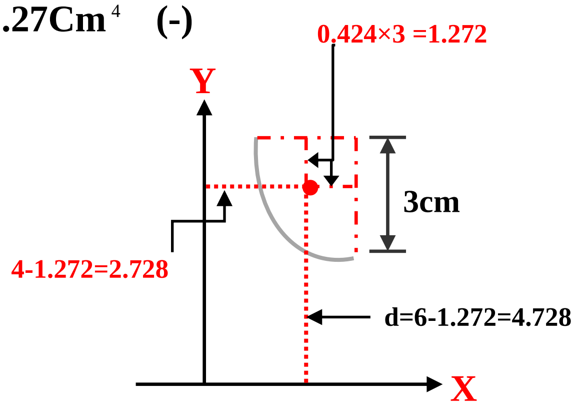


A2

For (A_3) :

$Ix = I_{\bar{x}} + Ad^2$

$I_x = 0.055r^4 + Ad^2$ **$= 0.055\times (3)^4 + 7.06\times (4.728)^2 = 162.27Cm^4$ (-)**



A3

I_x (total) $=288+54-162.27=179.73Cm^4$

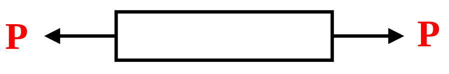
STRENGTH OF MATERIALS

Deals with relations between external loads and their internal effects on bodies.

STRESS: σ

Is the unit strength of a material and can be calculated by:

$$\boxed{\sigma = \frac{P}{A}}$$



P: axial force
A: cross sectional area

Units of stress: **N/m^2= Pa. (Pascal)**
MPa. =Mega Pascal =10^6 Pa. = N/mm^2

Types of stresses:

1:- Tensile stress

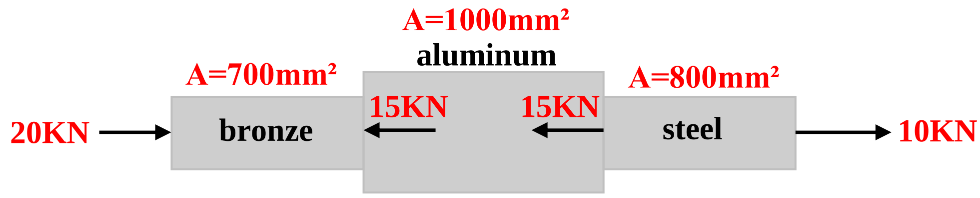
2:- Compressive stress

Example: An aluminum bar of (40mm) diameter carries an axial load of (12560N). Determine the stress in the bar.

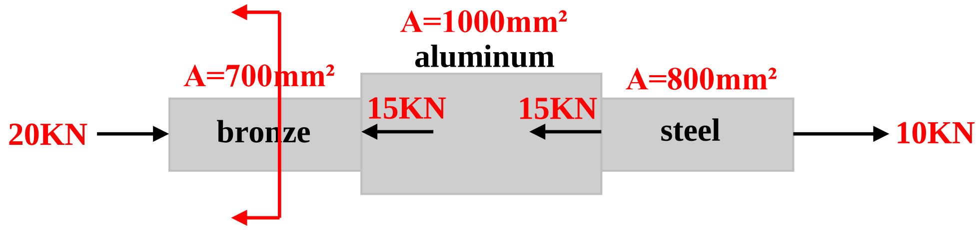
Solution:

σ =P/A
r=40/2=20mm
Cross sectional area (A)= $\pi \times r^2 = \pi \times (20/1000)^2 = \pi \times 400/10^6 = 1256\times10^{-6}$ m^2
σ =$12560/1256\times10^{-6}$ =10×10^6 pa. =10 MPa.

Example: An aluminum tube is rigidly fastened between a bronze bar and a steel bar. Axial loads are applied as shown in figure. Determine the stress in each material.



Solution:

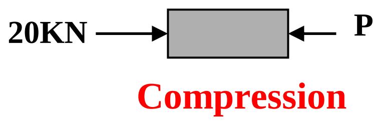


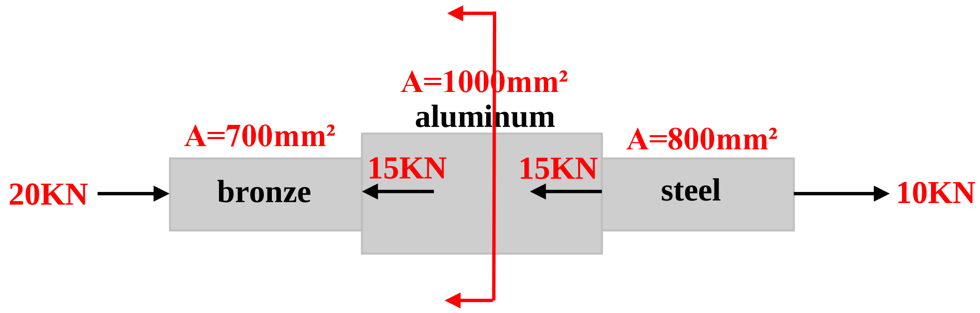
σ =P/A

$P_b = \sum Fx = 20$ KN

$\sigma_b = 20 \times 1000 / 700 \times 10^{-6}$

$= 28.57 \times 10^6 = 28.57$ MPa. (C)



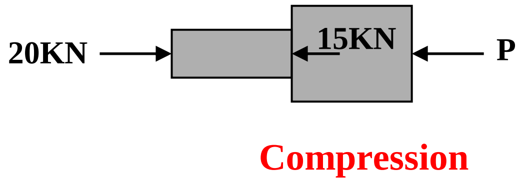


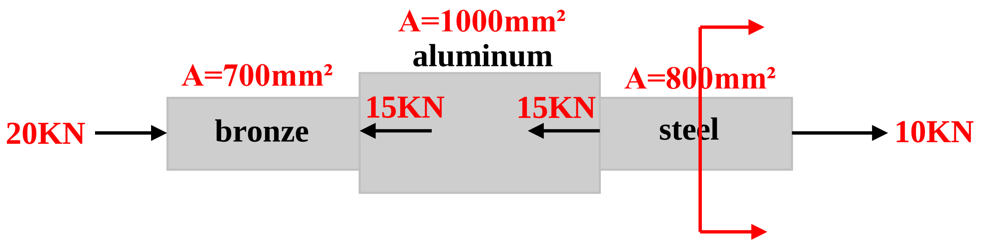
σ =P/A

$P_a = \sum Fx = 20 - 15 = 5$KN

$\sigma_a = 5 \times 1000 / 1000 \times 10^{-6}$

$= 5 \times 10^6$ pa =5 MPa. (C)



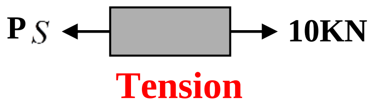


σ =P/A

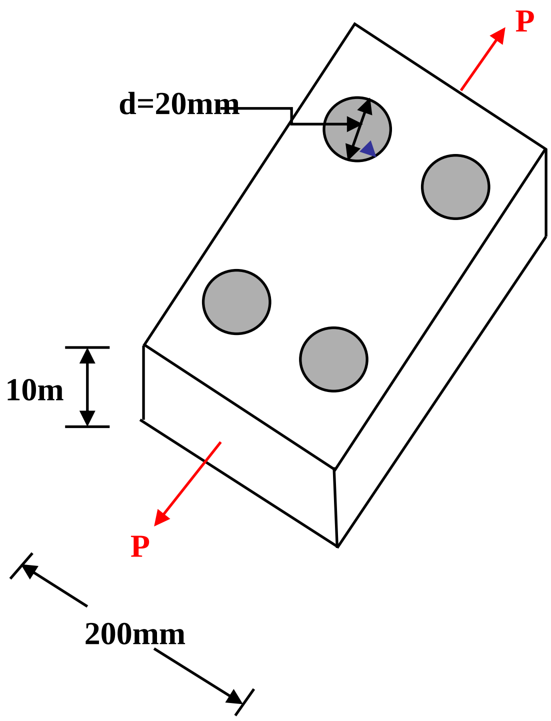
P_S =∑Fx=10KN

σ_S =10×1000/800×10^{-6}

=12.5× 10^6 pa =12.5 MPa. (T)



Example: Determine the maximum safe load (P) which may be applied on the steel plate shown in figure if the average tensile stress is (160 MPa.).



Solution:

$\sigma = P/A$

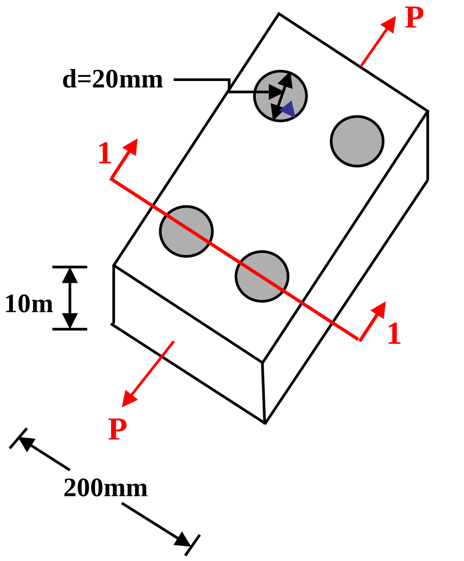
Area of section $= 200 \times 10 = 2000 mm^2$

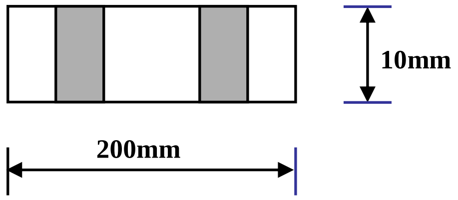
Area of two holes $= 2 \times 20 \times 10 = 400 mm^2$

Net area of section $= 2000 - 400$

$= 1600\ mm^2$

$= 1600 \times 10^{-6}\ m^2$





Section 1-1

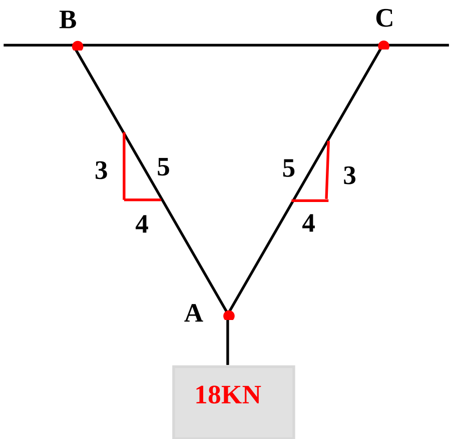
$\sigma = P/A$

$P = A \times \sigma$

$= 1600 \times 10^{-6} \times 160 \times 10^{6}$

$= 256000 N = 256 KN$

Example: A (18KN) weight is supported by two steel wires as shown in figure .Determine the cross sectional area of each wire if the tensile stresses in the wires are limited to (100 MPa.) .



Solution:

$\sum Fx=0$

$T_2\times4/5-T_1\times4/5=0$

$T_1=T_2=T$

$\sum Fy=0$

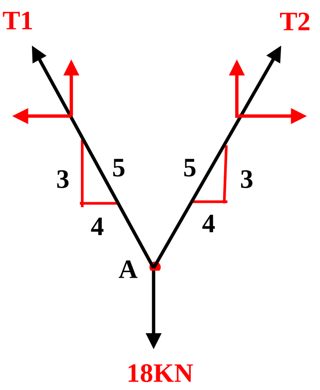
$T\times3/5+T\times3/5-18=0$

$T\times6/5=18$

$6T/5=18$

$6T=90$

$T=90/6=15KN$



$\sigma=P/A$

$A=P/\sigma$

$=15\times1000/100\times10^6$

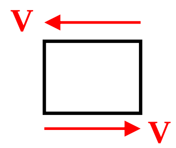
$=150\times10^{-6}\ m^2$

$=150mm^2$

3:-**Shearing stress:** τ

it is caused by a force acting parallel to area resisting the force.

$$\tau=\frac{V}{A}$$



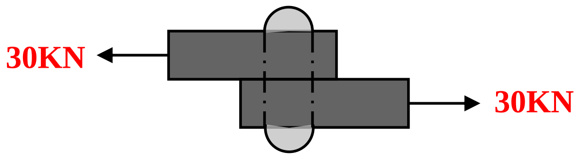
V: shearing force

A: area of parallel cross section

Example: Determine the shearing stress in the rivet shown in figure due to the (30KN) applying load if the diameter of the rivet is (20mm).

Solution

$$\tau=\frac{V}{A}$$



$V=30KN=30\times1000=30000N$

$d=20+1.5=21.5mm$

$r=21.5/2=10.75mm$

$A=\pi\times(10.75)^2=363\ mm^2=362.86\times10^{-6}\ m^2$

$$\tau=\frac{V}{A}$$

$=30000/362.86\times10^{-6}=82.67\times10^6$ Pa.= 82.67 MPa.

4:-**Bearing stress:**

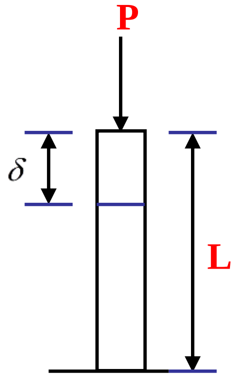
Is a contact pressure between separate bodies such as the soil pressure, force on bearing plate.

STRAIN: ε

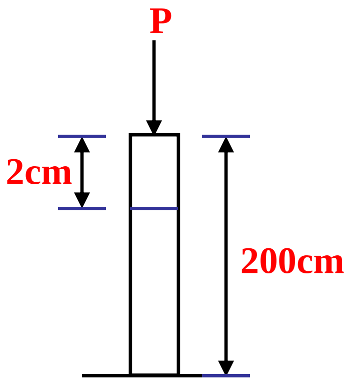
Is the unit deformation caused by stress

Strain = Change in length / Original length

$$\varepsilon = \frac{\delta}{L}$$



Example: Determine the strain of a body caused by the applied force (P) if the decrease in length is (2cm) , and the length of the body is (200cm) .



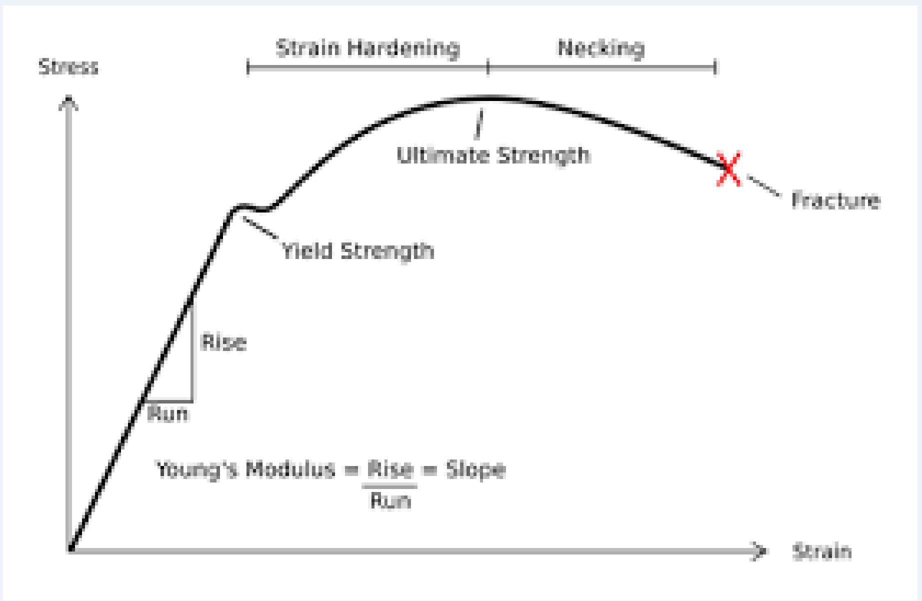
Solution:

$\varepsilon = \delta / L$

$= 2/200 = 0.01$

HOOK'S LAW: *Axial deformation*

The slope of stress-strain curve (straight line portion) =modulus of Elasticity = E



$E = \sigma / \varepsilon \Longrightarrow \sigma = E * \varepsilon$

NOTE: **the units of (E) are the same units of stress, for example:**

E for steel =200×10^9 Pa. =200GPa.
E for aluminum=70×10^9 Pa. =70 GPa.
GPa. =Gega Pascal= 10^9 Pa.

$$\sigma = E * \varepsilon$$

$$P/A = E * \delta / L$$

$$\delta = \frac{PL}{AE}$$

POISSON'S RATIO: ν

ν = **Lateral strain / Longitudinal strain**

$$\nu = \varepsilon_y / \varepsilon_x$$

Example: **A steel wire (8m) long hanging vertically support a tensile load of (4000N). Determine the required diameter and the elongation in the wire if the stress is not exceed (50Mpa.) . Assume Es=200Gpa.**

Solution:

$\sigma = P/A$

$50 \times 10^{6} = 4000/A$

$A = 4000/50 \times 10^{6} = 80 \times 10^{-6}\ m^2 = 80\ mm^2$

$A = \pi r^2$

$80 = \pi r^2$

$r^2 = 80/\pi = 25.46\ mm^2$

$r = 5.05\ mm$

$d = 5.05 \times 2 = 10.1\ mm$

$$\delta = \frac{PL}{AE}$$

$= 4000 \times 8 / 80 \times 10^{-6} \times 200 \times 10^{9}$

$= 2 \times 10^{-3}\ m$

$= 2\ mm$

Another solution :

$E = \sigma / \varepsilon$

$200 \times 10^{9} = 50 \times 10^{6} / \varepsilon$

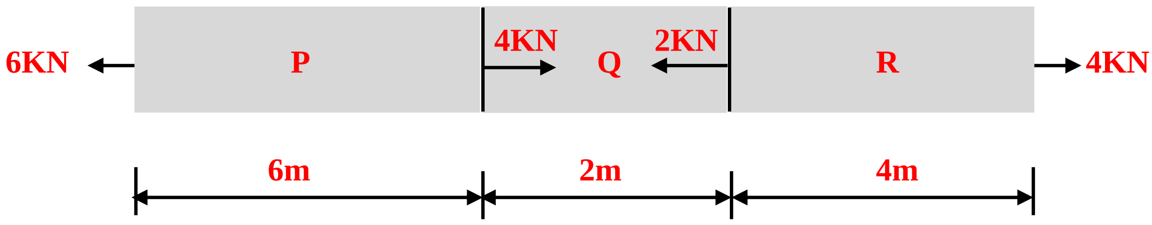
$\varepsilon = 50 \times 10^{6} / 200 \times 10^{9} = 0.25 \times 10^{-3}$

$\varepsilon = \delta / L$

$0.25 \times 10^{-3} = \delta / 8 \times 10^{3}$

$\delta = 0.25 \times 10^{-3} \times 8 \times 10^{3} = 2\ mm$

Example: A uniformly bar of (1×10^{-4} m^2) area. Axial loads are applied as shown in figure. Find the total deformation. Assume (E=200GPa.).

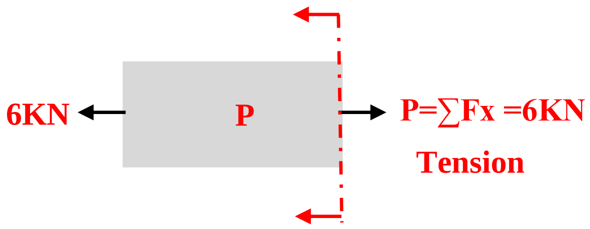


Solution:

$$\delta = \frac{PL}{AE}$$

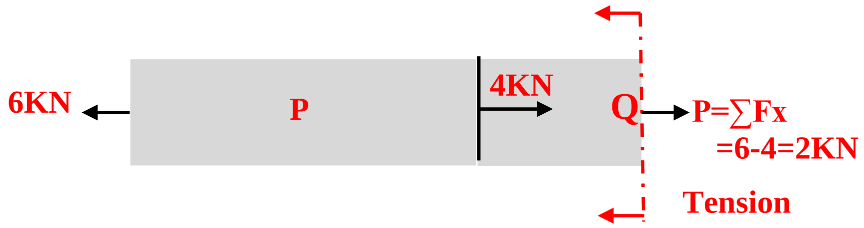
δ_P $=6\times1000\times6/1\times10^{-4}\times200\times10^{9}$

$=0.0018m=1.8mm$ (+)



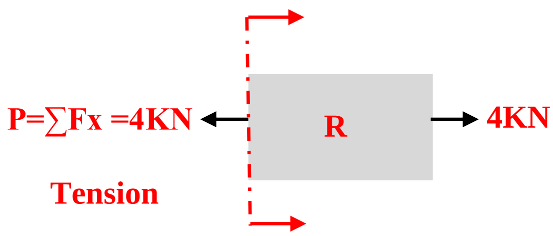
δ_Q $=2\times1000\times2/\ 1\times10^{-4}\times200\times10^{9}$

$=0.0002m=0.2mm$ (+)



δ_R $=4\times1000\times4/1\times10^{-4}\times200\times10^{9}$

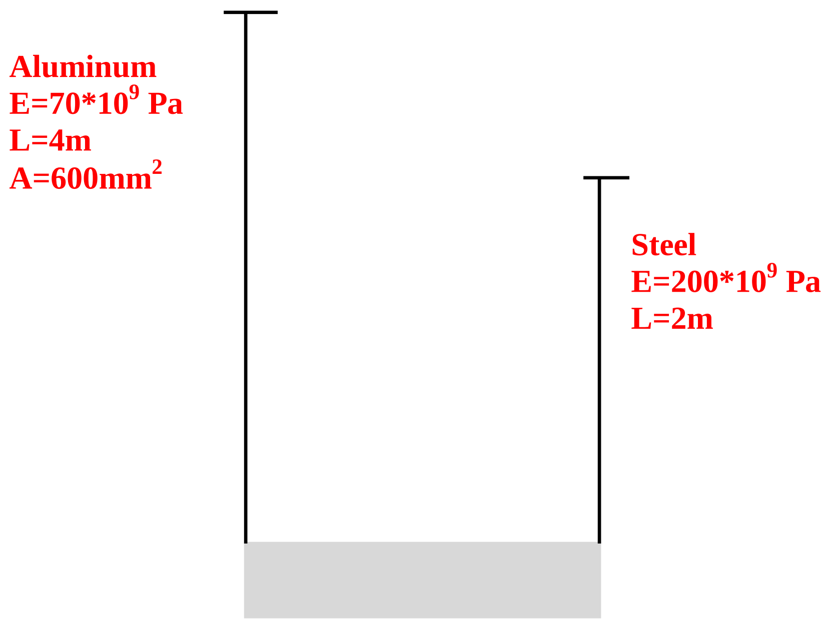
$=0.0008m=0.8mm$ (+)



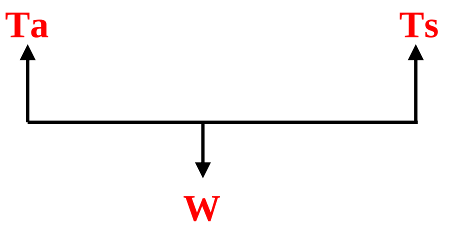
δ total=1.8+0.2+0.8

=2.8mm

Example: uniform concrete beam of mass (1200kg) will remain level after it is attached to a steel and aluminum rods as shown in figure. Determine:
1-the vertical movement of the beam.
2-the cross sectional area of steel rod.



Solution:



$$\delta_s = \frac{PL}{AE}$$

Weight of beam=1200×9.8=11760N

$$\textbf{Ts = Ta} = \frac{11760}{2} = 5880N$$

$$\delta = \frac{PL}{AE} = \frac{5880 \times 4}{600 \times 10^{-6} \times 70 \times 10^{9}} = 0.56 \times 10^{-3} m = 0.56 mm$$

The beam will remain level after it is attached, then

$$\delta_s = \delta_a$$

$$\delta_s = \frac{PL}{AE}$$

$$0.56 \times 10^{-3} = \frac{5880 \times 2}{A \times 200 \times 10^{9}}$$

$$A = \frac{5880 \times 2}{0.56 \times 10^{-3} \times 200 \times 10^{9}} = 105 \times 10^{-6} m^2 = 105 mm^2$$

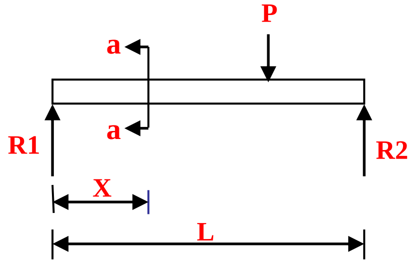
SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR BEAMS

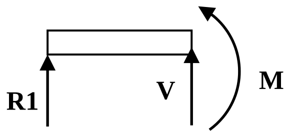
Shear force: is the summation of vertical external loads acting on the left side of the selected section.

Bending moment: is the summation of moments of all the loads acting to the left of the selected section.

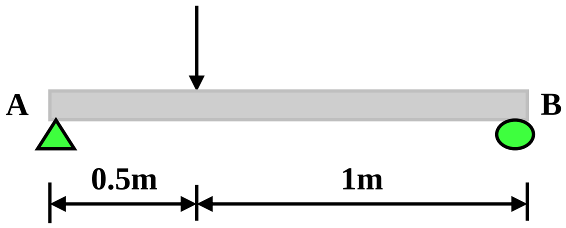
$$V = (\sum Fy)_L$$

$$M = (\sum M)_L$$





Example: Draw shear force and bending moment diagrams for the beam loaded as shown in figure.



Solution:

1-determination of reactions

$\sum Fx=0$ $\quad$ $Ax=0$

$+\circlearrowright$ $\sum M_A=0$

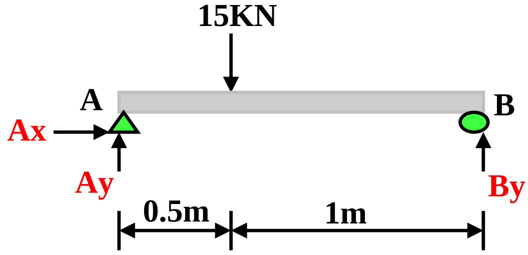
$By\times1.5-15\times0.5=0$

$By=5KN$

$\sum Fy=0$

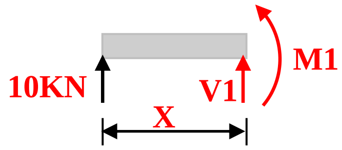
$Ay+5-15=0$

$Ay=10KN$



2-Drawing of S.F.D and B.M.D written the equations at section (1-1) and (2-2).

Section (1-1)



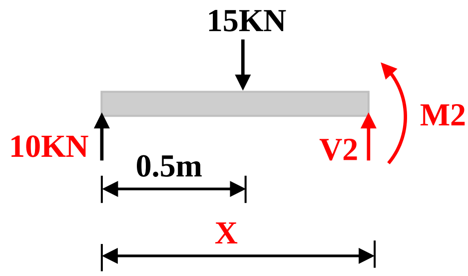
$V1=\sum F_y=10KN$

$+ \curvearrowright$ $M1=\sum M=10X$

When $X=0$ $M1=0$

When $X=0.5$ $M1=5KN.m$

Section (2-2)

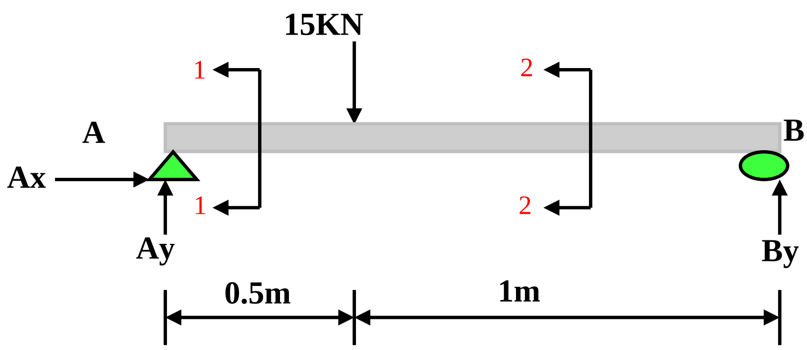


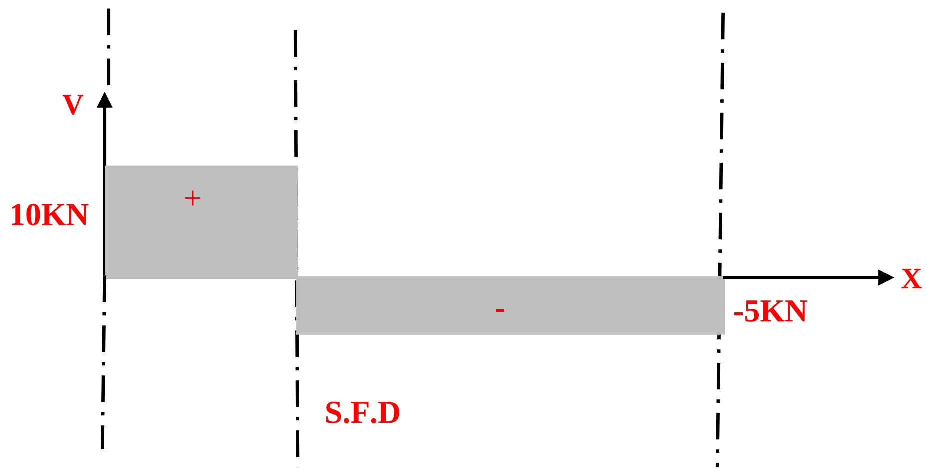
$V_2=\sum F_y=10-15=-5KN$

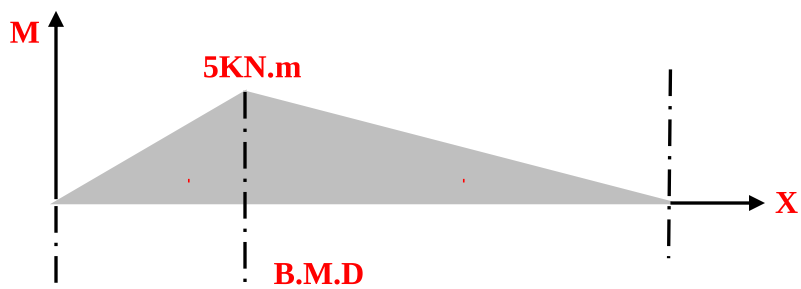
$+ \curvearrowright$ $M_2=\sum M=10X-15(X-0.5)$

When $X=0.5$ $M2=10*0.5-15*(0.5-0.5)=5KN.m$

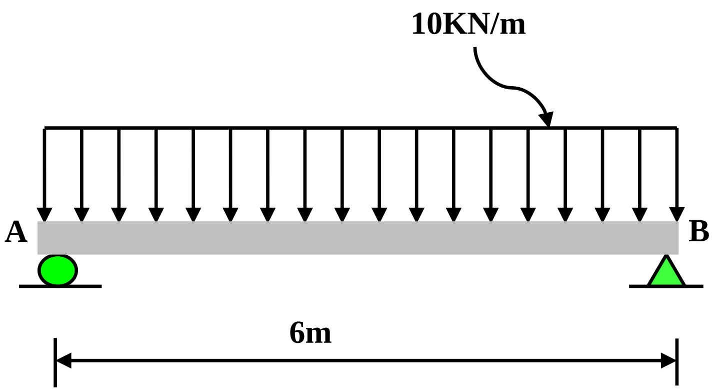
When $X=1.5$ $M2=10*1.5-15*(1.5-0.5)=0$







Example: Draw shear force and bending moment diagrams for the beam loaded as shown in figure .



Solution:

1-determination of reactions

R=10× 6=60KN

∑Fx=0 Bx=0

+↺ $\sum M_A=0$

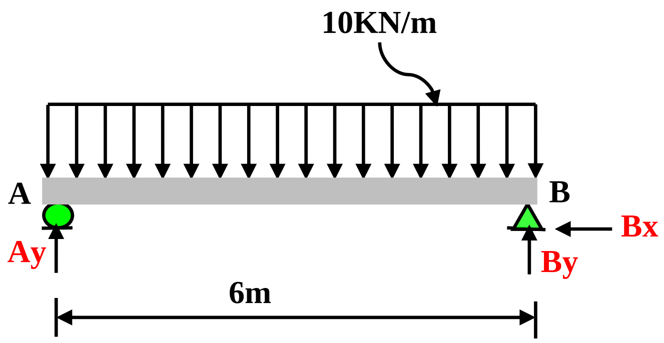
By×6-60×3=0

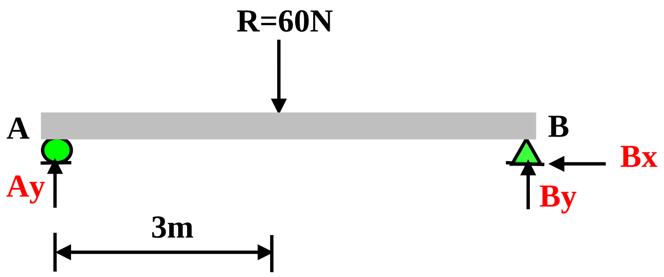
By=30KN

∑Fy=0

Ay+30-60=0

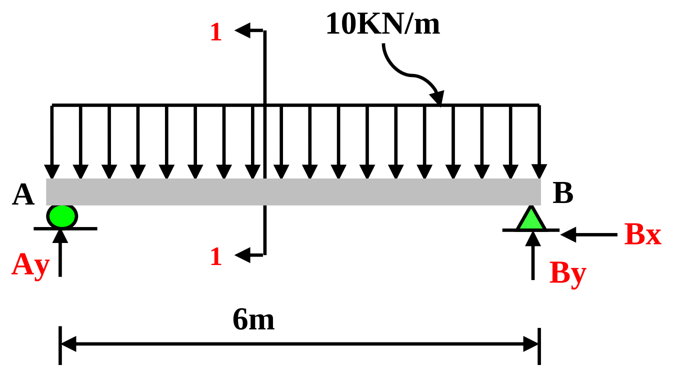
Ay=30KN



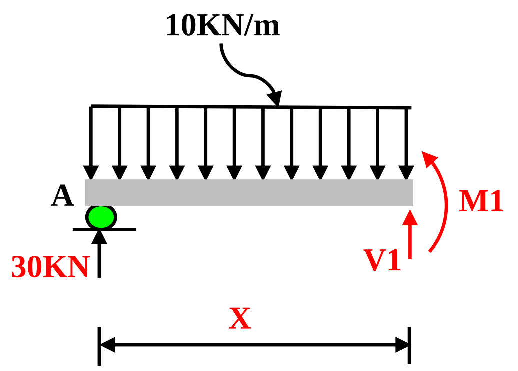


F.B.D

2-Drawing of S.F.D and B.M.D by written the equations at section (1-1)



Section (1-1)



R=10X

V=∑Fy=30-10X

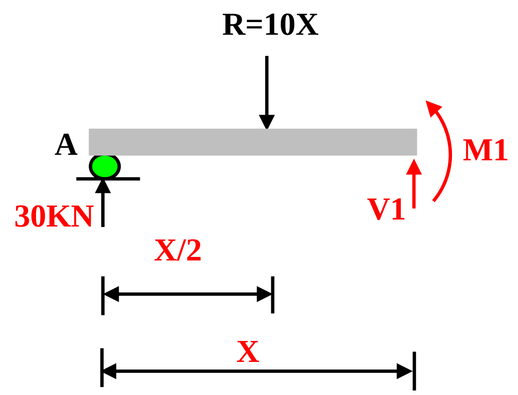
when X=0 V1=30KN

when X=6 V1= -30KN

+ ↷ M=∑M =30X-10X(X/2) =30X-5X²

when X=0 M1=0

when X=6 M1=0



F.B.D

Note: the maximum bending moment caused when V=0

therefore:

V=30-10X

0=30-10X

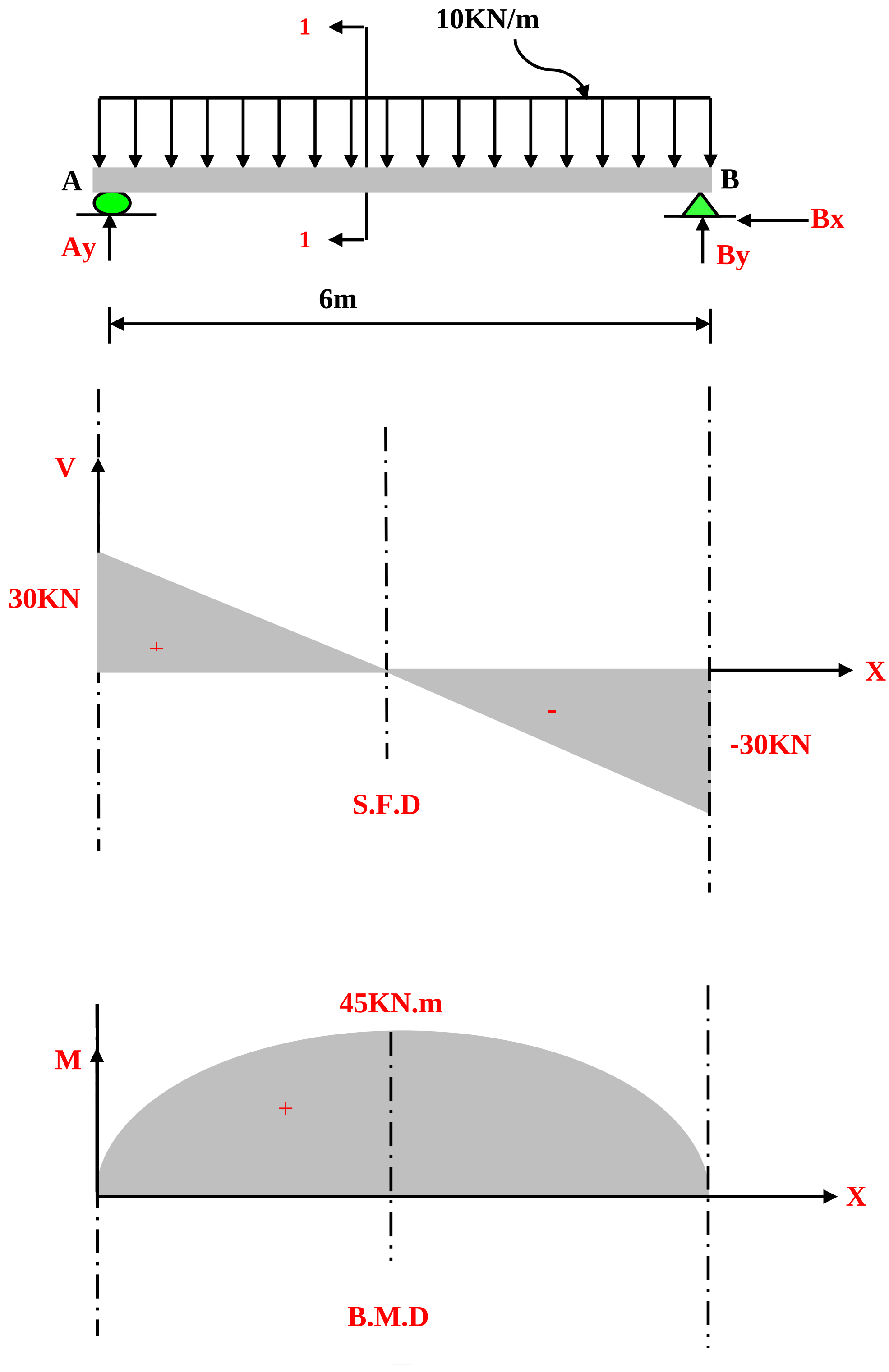
30=10X

X=30/10=3m

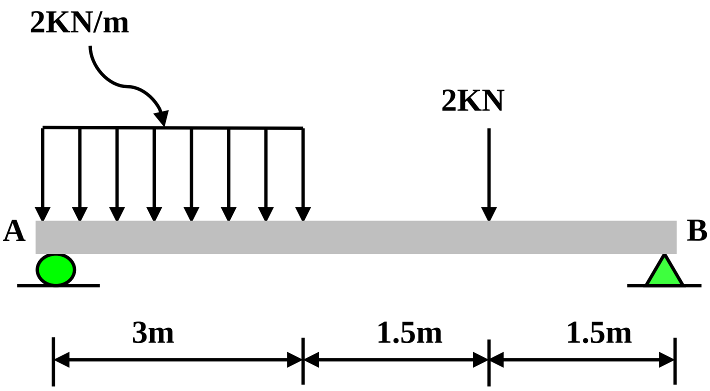
when X=3

Mmax.=30*3-5*3²

=90-45= 45KN.m

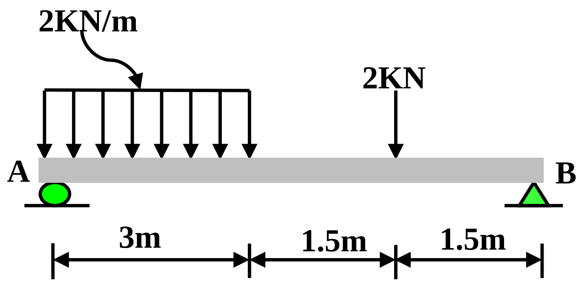

1
10KN/m
A
B
Ay
1
By
Bx
6m
V
30KN
+
-
X
-30KN
S.F.D
45KN.m
M
+
X
B.M.D

Example: Draw shear force and bending moment diagrams for the beam loaded as shown in figure.



Solution:

1-determination of reactions



$R=3\times 2=6KN$

$\sum Fx=0$ **$Bx=0$**

$+\circlearrowright \sum M_A=0$

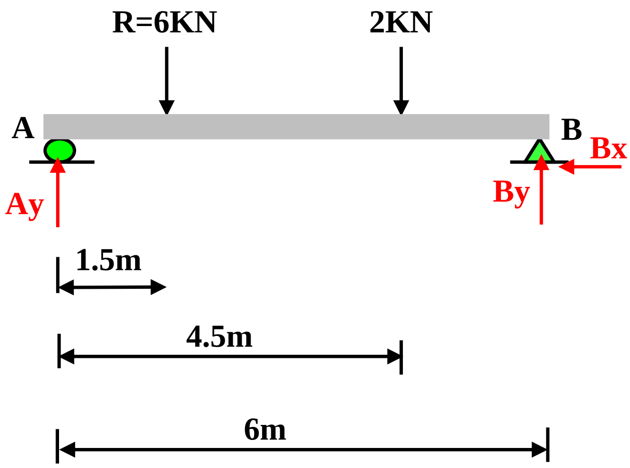
$By\times 6-2\times 4.5-6\times 1.5=0$

$By=3KN$

$\sum Fy=0$

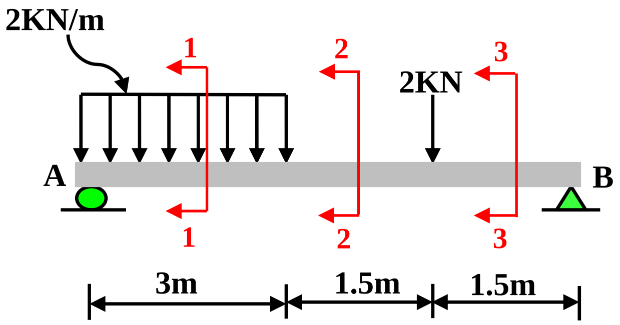
$Ay+3-6-2=0$

$Ay=5KN$



F.B.D

2-Drawing of S.F.D and B.M.D by written the equations at sections (1-1) , (2-2) and (3-3).



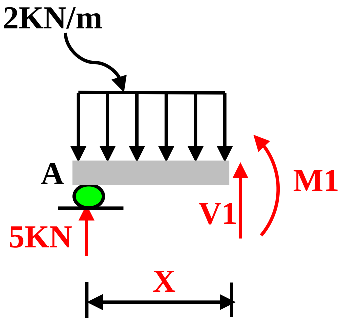
Section (1-1)

R=2X

V1=∑Fy= 5-2X

when X=0   V1=5KN

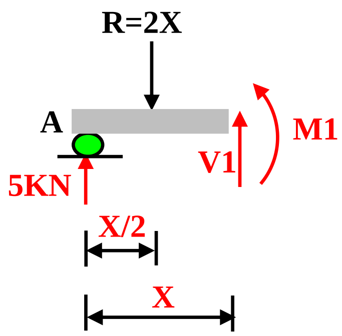
when X=3   V1= 5 -(2* 3)= -1KN



+ ↻ M1=∑M =5X-2X(X/2) =5X-X²

when X=0   M1=0

when X=3   M1=(5*3)-3²=6KN.m



Note: the maximum bending moment caused when V=0

therefore:

V1=5-2X

0=5-2X

5=2X

X=5/2=2.5m

when X=2.5

Mmax.= 5X-X²

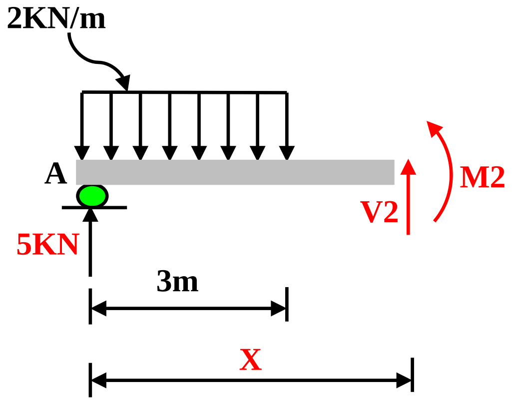
=5*2.5-2.5²= 6.25KN.m

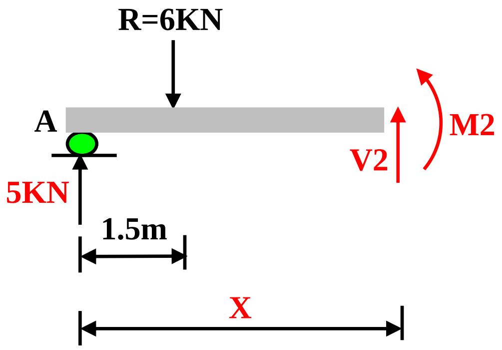
Section (2-2)

R=2× 3=6KN
V2=∑Fy=5-6=-1KN

$+ \curvearrowright$ M2=∑M =5X-6(X-1.5)
=5X-6X+ 9
=9-X

when X=3 M2=9-3=6KN.m
when X=4.5 M2=9-4.5=4.5KN.m



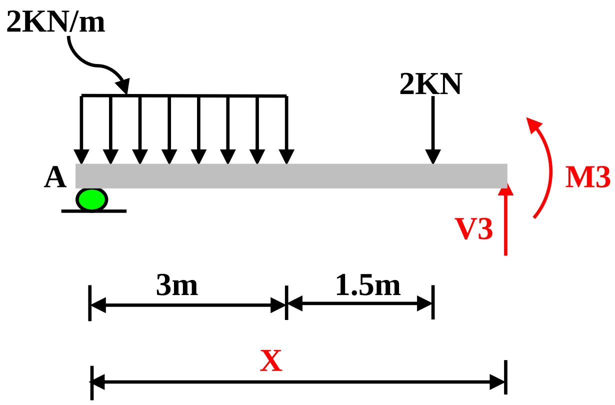


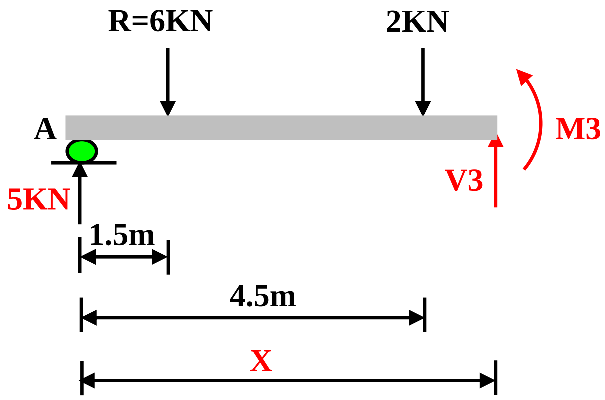
Section (3-3)

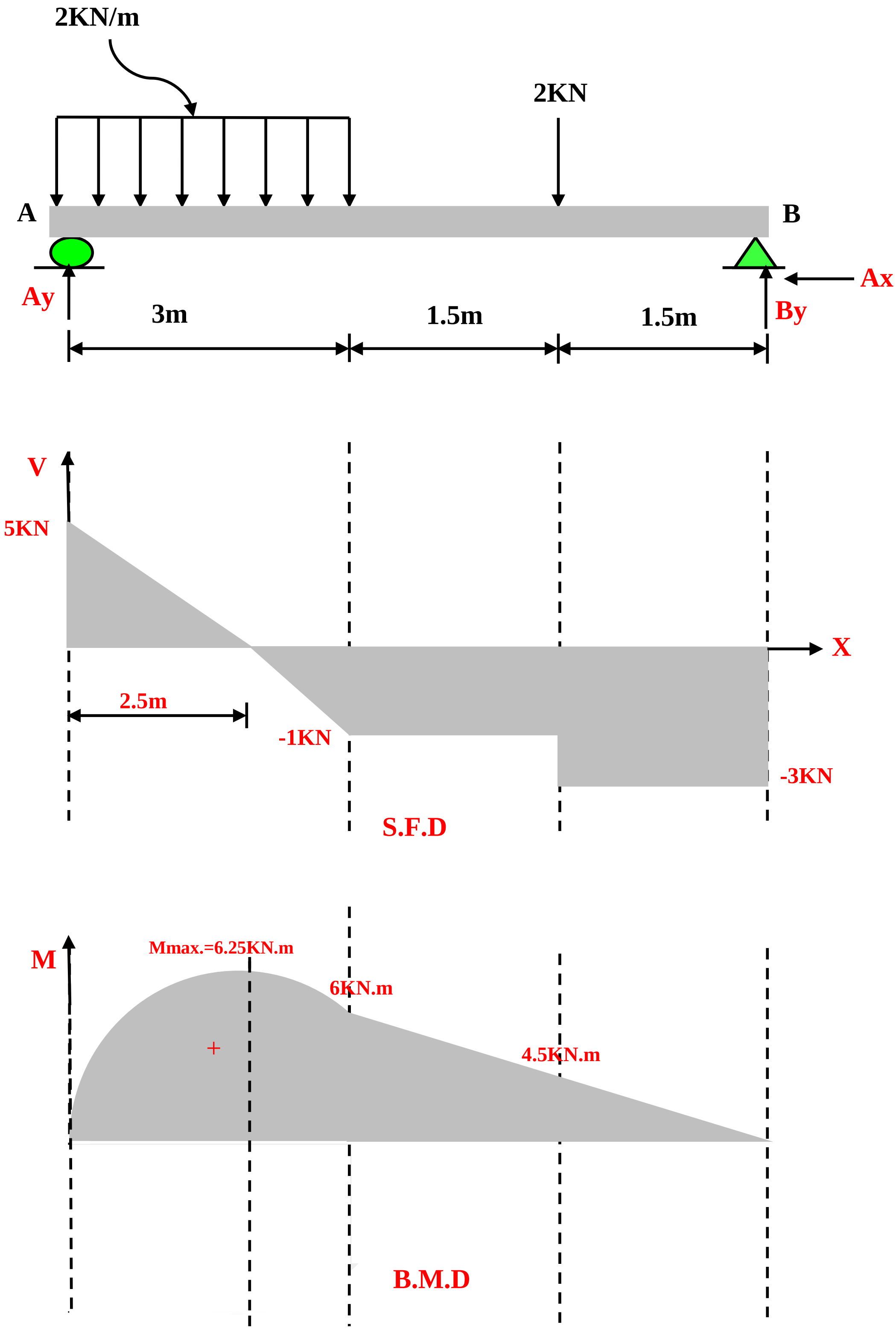
R=2× 3=6KN
V3=∑Fy=5-6-2=-3KN

$+ \curvearrowright$ M3=∑M =5X-6(X-1.5)-2(X-4.5)
=5X-6X+9-2X+9
=18-3X

when X=4.5 M3=18-3×4.5=4.5KN.m
when X=6 M3=18-3×6=0KN.m





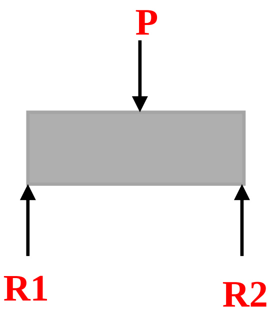

2KN/m
2KN
A
B
Ax
Ay
By
3m
1.5m
1.5m
V
5KN
X
2.5m
-1KN
-3KN
S.F.D
M
Mmax.=6.25KN.m
6KN.m
4.5KN.m
+
B.M.D

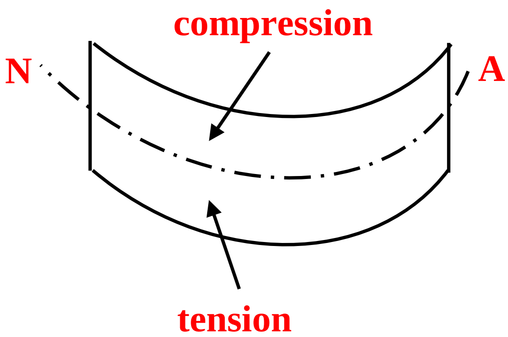
STRESSES IN BEAMS: (Rectangular sections)

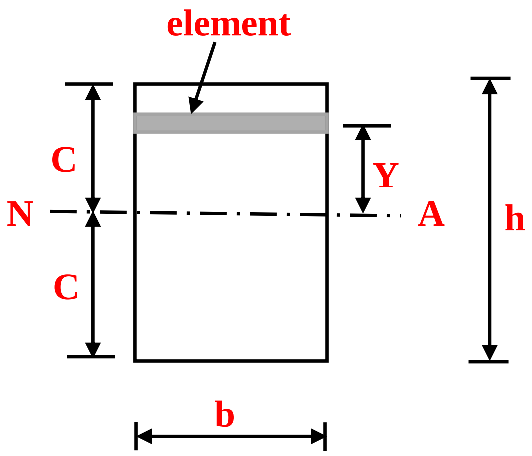
1:-Bending stress: (Flexure stress)

Is the stress caused by the bending moment.

Flexure formula: **is the relation between bending stress and the bending moment.**







$$\sigma = \frac{MY}{I}$$

σ =flexure stress (N/m²) at a distance Y from N.A

Y=distance from N.A to element

M=maximum bending moment at the section

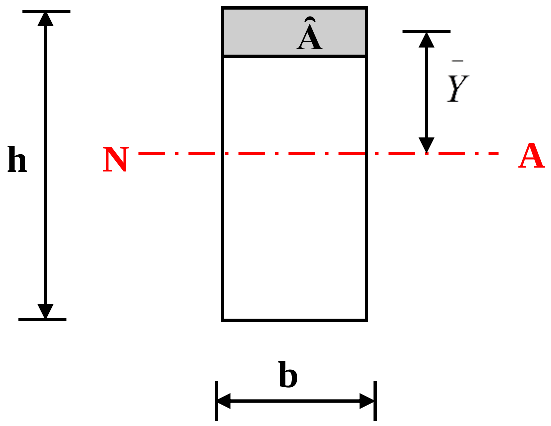
I =moment of inertia of the section = $bh^3/12$

$$\sigma\, max. = \frac{MC}{I}$$

$\sigma_{max.}$ =maximum flexure stress

C =the distance from N.A to the top or bottom of the section = h / 2

2:-Shearing stress: τ



$$\tau = V \hat{A} \bar{Y} / I\, b$$

Â =shaded area

$\bar{Y}$ =distance from centroid of Â to the N.A

V =maximum shearing force

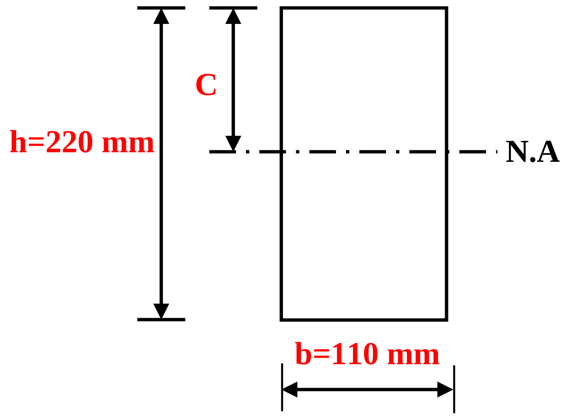
$$\tau\, max. = \frac{3V}{2A}$$

A= b*h

Example: A cantilever beam (110mm) wide by (220mm) height if $M_{max.}$=10.75KN.m , and $V_{max.}$=8KN. Determine:-

1- the maximum flexure stress
2- the maximum shear stress

Solution:



$$\sigma\, max. = \frac{MC}{I}$$

C = 220/ 2 = 110mm =0.11m

$I = bh^3/12 = (110*10^{-3})*(220*10^{-3})^3/12$
$= 110*10^{-3} * 10648000 * 10^{-9} /12$
$= 110*10^{-3} * 10648*10^{3} * 10^{-9} /12$
$= 97606.66 * 10^{-9}$
$= 97.606 * 10^{3} * 10^{-9} = 97.606 * 10^{-6}\ m^4$

$$\sigma\, max. = \frac{10.75 \times 1000 \times 0.11}{97.606 * 10^{-6}} = 12.11 * 10^{6}\ pa. = 12.11\ Mpa.$$

$$\tau\ max. = \frac{3V}{2A}$$

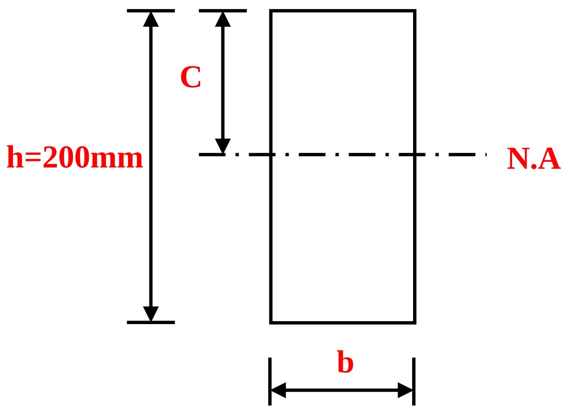
A= b × h = (110 * 10^{-3}) × (220 * 10^{-3}) = 24200 × 10^{-6}

$$\tau\ max. = \frac{3\times 8\times 1000}{2\times\ 24200\ \ \times 10-6}$$ = 0.49 × 10^{6} Pa. = 0.49 MPa.

Example: Determine the minimum width **(b)** of a beam if the flexural stress is not exceed **(12×10^{6}Pa.)**, **($M_{max.}$ =5000N.m)** and its height **(h=200mm).**

Solution:

$$\sigma\ max. = \frac{MC}{I}$$



C = 200/ 2 = 100mm = 0.1m

$$\sigma\ max. = \frac{MC}{I}$$

12 * 10^{6} = 5000 * 0.1/ I

I= 5000 * 0.1 / 12 * 10^{6} = 41.66 *$10^{-6}\ m^{4}$

I = bh^{3}/12

h= (200 / 10^{3}) = 200 * 10^{-3} m

41.66 *10^{-6} = b*(200 * 10^{-3})^{3}/12

41.66 *10^{-6} = b * 8000000 * 10^{-9} / 12

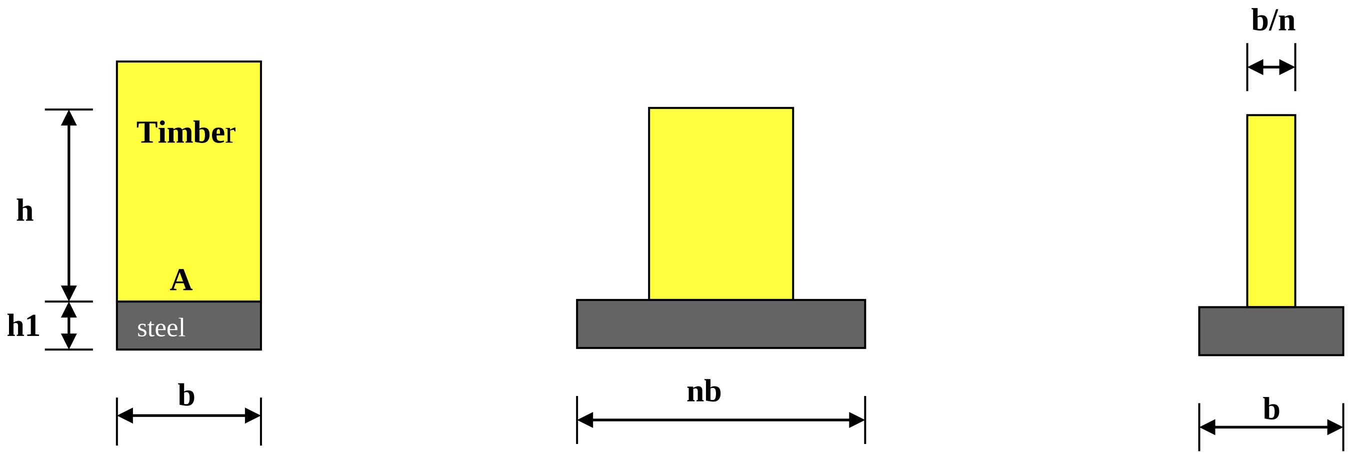
41.66 *10^{-6} * 12 = b * 8 * 10^{6} * 10^{-9}

b = 41.66 *10^{-6} * 12 / 8 * 10^{6} * 10^{-9}

b= 62.49 * 10^{-3} m = 62.49 mm

COMPOSITE BEAMS: *(Beams of different materials)*

The most common method of dealing with a non- homogenous beam is to transform it into an equivalent homogenous beam .



a) timber and steel section **b) equivalent wood section** **c) equivalent steel section**

strain of steel = strain of wood (at point A)

$$\varepsilon_S = \varepsilon_W$$

$$\sigma_s / E_S = \sigma_W / E_W \quad \text{----------- (1)}$$

$$P_S = P_W$$

$$A_S \, \sigma_S = A_W \, \sigma_W \quad \text{--------- (2)}$$

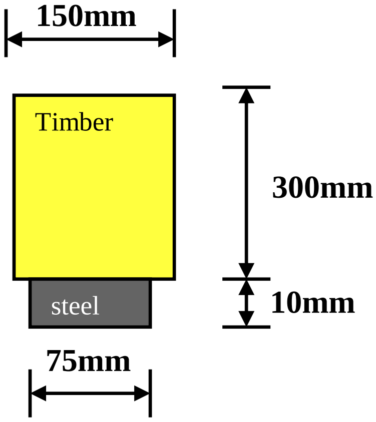
From eq.(1) and eq.(2)

$$A_S \, (E_S / E_W) \, \sigma_W = A_W \, \sigma_W$$

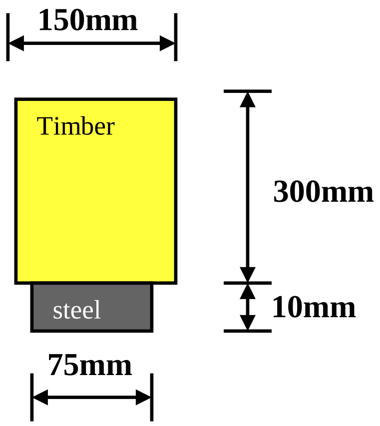
$$A_W = \mathbf{n} \, A_S$$

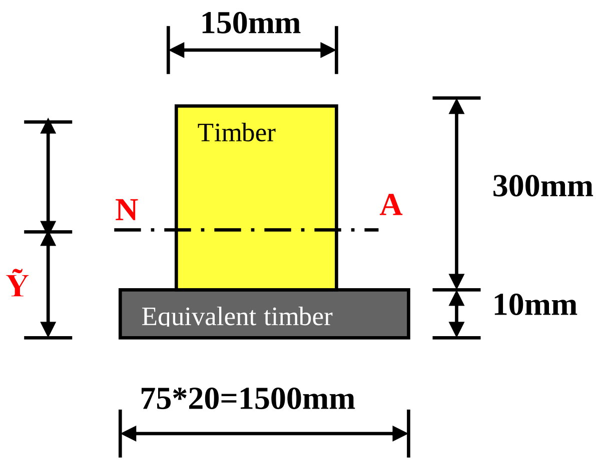
$$\mathbf{n} = E_S / E_W$$

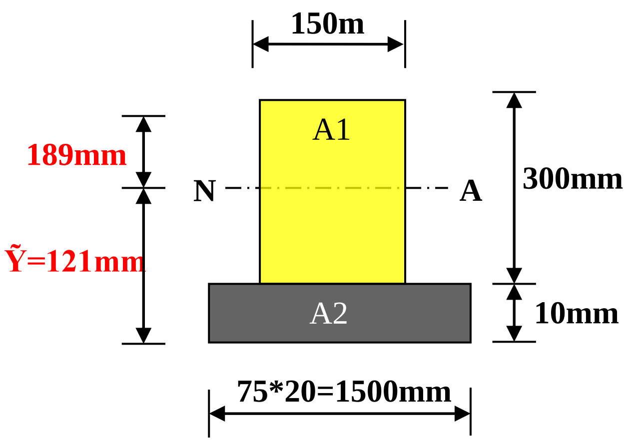
Example : A timber beam (150mm) by (300mm) is reinforced on the bottom only with a steel strip (75mm) wide by (10mm) thick as shown in figure . Determine the maximum resisting moment if the allowable stresses are $\sigma_s \leq 120$Mpa. , and $\sigma_w \leq 8$Mpa. .Assume (n=20) .



Solution :







A_{w1} =150*300=45000mm²

A_s =75*10= 750 mm²

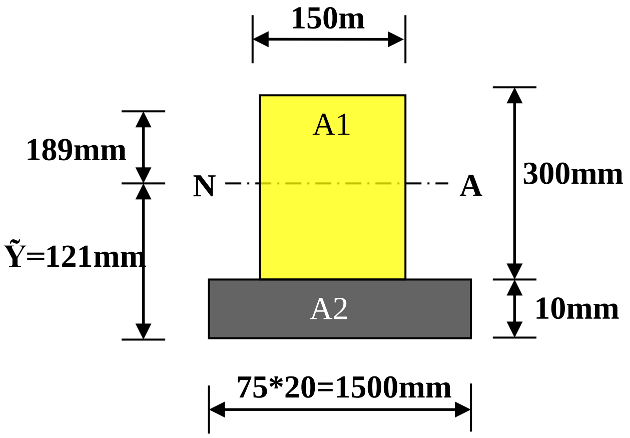
(A_W) equivalent for steel =n A_s =20*750=15000 mm²

Total equivalent wood area of section =60000 mm²

Location of N.A from the base of section:

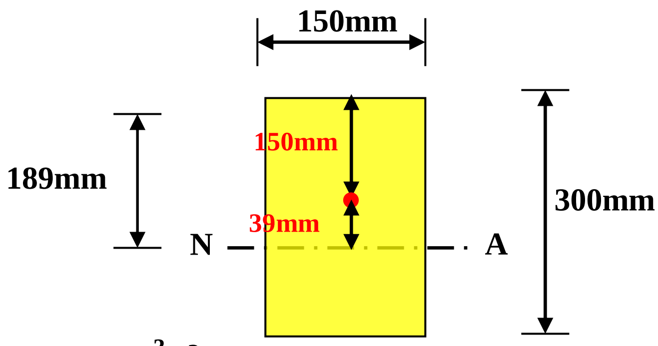
60000* Ỹ=45000*160+15000*5

Ỹ=121mm



A1=$(300\times10^{-3})\times(150\times10^{-3})=45000\times10^{-6}$ m^2
A2=$(1500\times10^{-3})\times(10\times10^{-3})=15000\times10^{-6}$ m^2

For (A1):



A1

$$I_{N.A} = I_{\bar{x}} + Ad^2$$

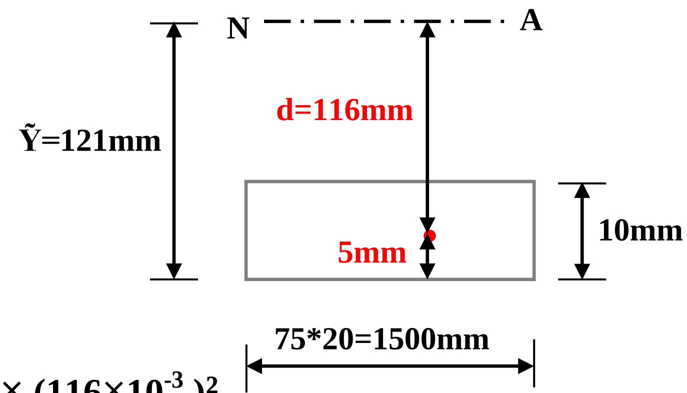
$$= \frac{bh^3}{12} + Ad^2$$

$$= (150\times10^{-3}) \times (300\times10^{-3})^3 / 12 + 45000\times10^{-6} \times (39\times10^{-3})^2$$

$$= 337500000 \times 10^{-12} + 68445000 \times 10^{-12}$$

$$= 405945000 \times 10^{-12} \text{ m}^4 \quad (+)$$

For (A2):



A2

$$I_{N.A} = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2$$

$$= (1500\times10^{-3}) \times (10\times10^{-3})^3 / 12 + 15000\times10^{-6} \times (116\times10^{-3})^2$$

$$= 125000 \times 10^{-12} + 201840000 \times 10^{-12}$$

$$= 201965000 \times 10^{-12} \text{ m}^4 \quad (+)$$

$$I_{N.A} \text{ (total)} = 271926500 \times 10^{-12} + 143987466.66 \times 10^{-12}$$

$$= 607910000 \times 10^{-12} \text{ m}^4$$

$$\sigma = \frac{MY}{I}$$

M=σ I/Y

$M_w = 8\times10^6 \times 607910000 \times 10^{-12} / 189 \times 10^{-3} = 25731640.21 \times 10^{-3}$ N.m

$= 25731640.21 \times 10^{-6}$ KN.m

= 25.73 KN.m

In wood equivalent of the steel:

$\sigma_W = \sigma_S$ /n=120/20=6Mpa.

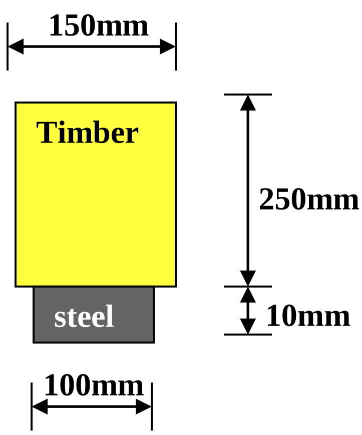
$M_S = 6\times10^6 \times 607910000 \times 10^{-12} / 121\times 10^{-3} = 30144297.52$ N.m

$= 30144297.52 \times 10^{-6}$ KN.m

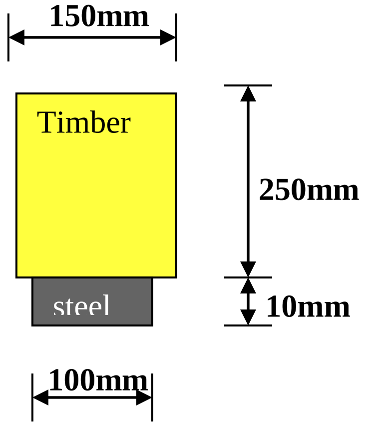
= 30.14 KN.m

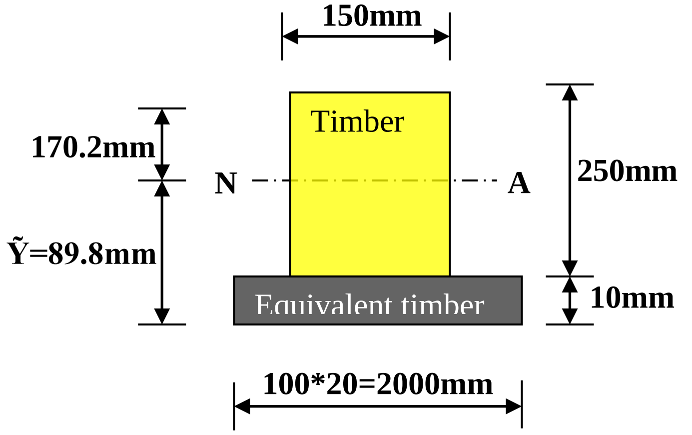
The smaller resisting moment M_w =25.73 KN.m is the safe resisting moment.

Example :A timber beam (150mm) by (250mm) is reinforced on the bottom only by a steel plate (100mm) wide by (10mm) thick .Determine the concentrated load that can be applied at the center of a simply supported span (6m) long, if (n=20), maximum $\sigma_S \leq$ 120Mpa., and $\sigma_W \leq$ 8Mpa. , show that the neutral axis (N.A) is (170.2mm) below the top and that $I_{N.A} = 416\times10^6$ mm ^{4}.



Solution :



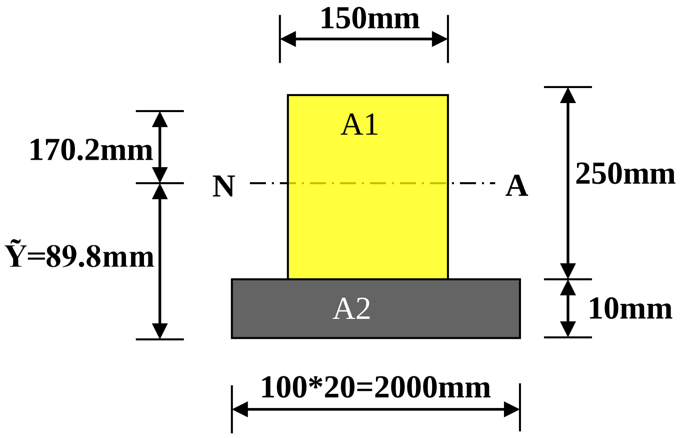


$$A\bar{Y} = \sum ay$$

$$(150*250 + 2000*10)*\bar{Y} = 250*150*135 + (2000*10*5)$$

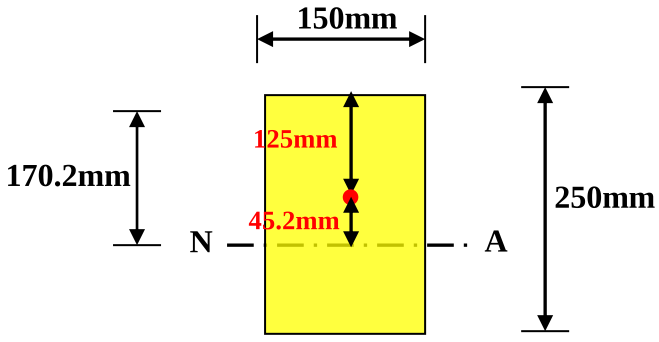
$$\bar{Y} = 89.8mm$$

260-89.8=170.2mm



A1=$(250\times10^{-3}) \times (150\times10^{-3}) = 37500\times10^{-6}\ m^2$
A2=$(2000\times10^{-3}) \times (10\times10^{-3}) = 20000\times10^{-6}\ m^2$

For (A1):



A1

$$I_{N.A} = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2$$

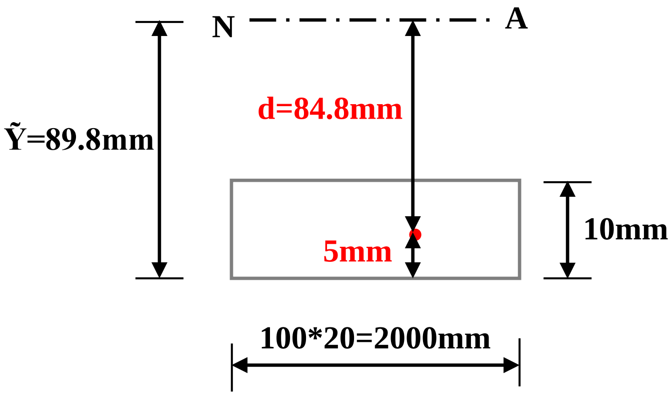
= (150×10^-3^) × (250×10^-3^)^3^ / 12 + 37500×10^-6^ × (45.2×10^-3^)^2^

$= (150\times10^{-3}) \times (250\times10^{-3})^3 / 12 + 37500\times10^{-6} \times (45.2\times10^{-3})^2$

$=195312500 \times 10^{-12} + 76614000 \times 10^{-12}$

$= 271926500 \times 10^{-12}$ m^4 (+)

For (A2):



A2

$$I_{N.A} = I_{\bar{x}} + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2$$

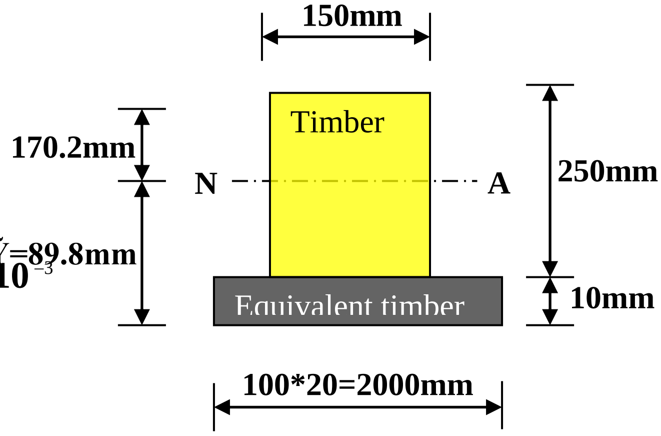
$= (2000\times10^{-3}) \times (10\times10^{-3})^3 / 12 + 20000\times10^{-6} \times (84.8\times10^{-3})^2$

$=166666.66 \times 10^{-12} + 143820800 \times 10^{-12}$

$= 143987466.66 \times 10^{-12}$ m^4 (+)

$I_{N.A}$ (total) $= 271926500 \times 10^{-12} + 143987466.66 \times 10^{-12} = 415913966.66 \times 10^{-12}$ m^4

$$\sigma = \frac{MY}{I}$$



M=σ I/Y

$M_w = 8*10^6 \times 415913966.66 \times 10^{-12}$ /$170.2*10^{-3}$

$= 19549422.63 \times 10^{-3}$ N.m

$= 19549422.63 \times 10^{-6}$ KN.m

=19.54 KN.m

In wood equivalent of the steel:

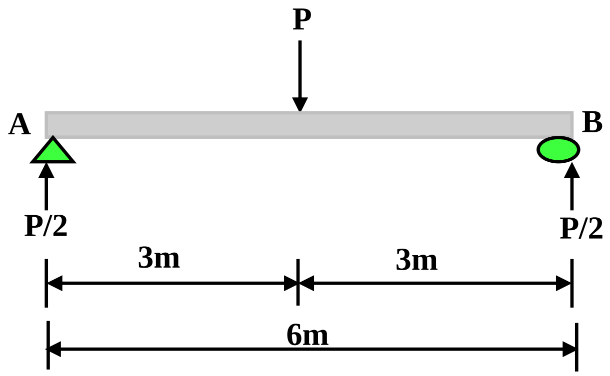
$\sigma_W = \sigma_S$/n=120/20=6MPa.

$M_S = 6*10^6 * 415913966.66 \times 10^{-12}$ / $89.8*10^{-3}$

$= 27789351 \times 10^{-3}$ N.m

$= 27789351 \times 10^{-6}$ KN.m

=27.78 KN.m



$$M = \frac{P}{2} \times 3$$

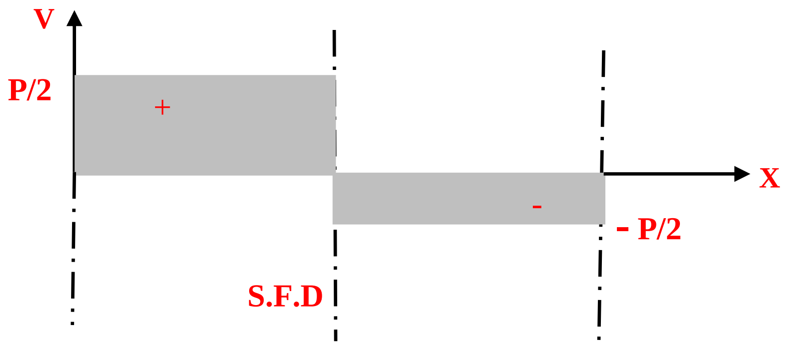
$$19.54 = \frac{P}{2} \times 3$$

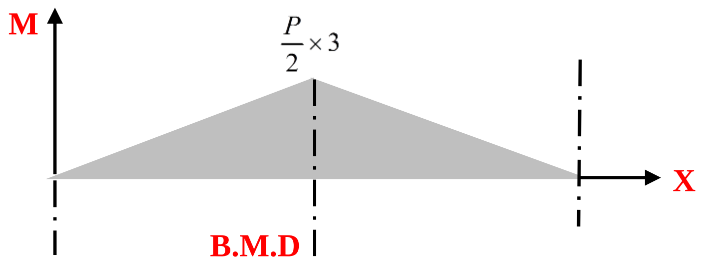
$$19.54 \times 2 = P \times 3$$

$$P = \frac{19.54 \times 2}{3}$$

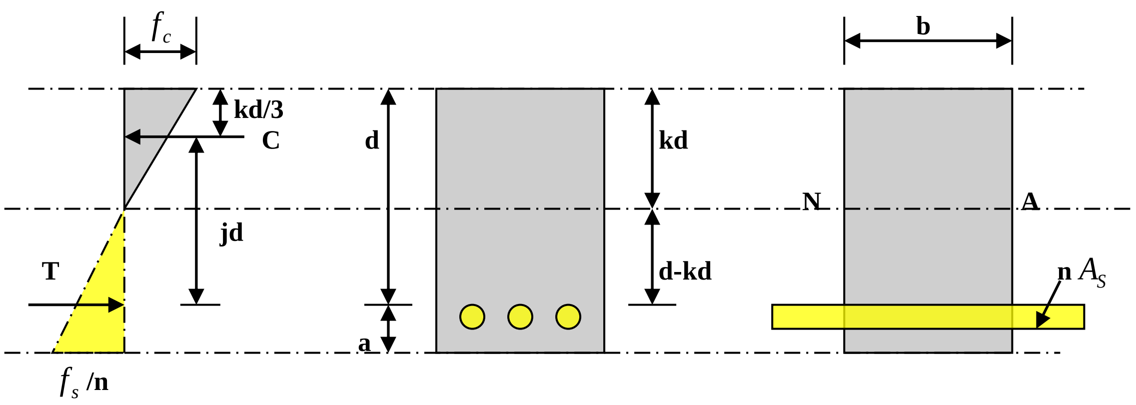
$$P = \frac{19.54 \times 2}{3}$$

$$P = 13.02 KN$$





REINFORCED CONCRETE BEAMS



d :the distance from the top of the beam to the center of the reinforcing steel (effective depth)
kd: the distance from the top of the beam to N.A

NOTE: the N.A is located by applying the principles that the moment of area above the N.A is equal the moment of the area below this axis .

$$(b*kd)(kd/2) = n A_S (d- kd)$$

NOTE: the resultant compressive force (C) in concrete acts at distance (kd/3) from the top of the beam .

$$M_C = 1/2 * f_c (bkd)(jd)$$

$$M_S = f_s A_S (jd)$$

C :compressive force in concrete
T :tensile force in steel
f_c :maximum compressive stress in concrete
f_s :the tensile stress in steel

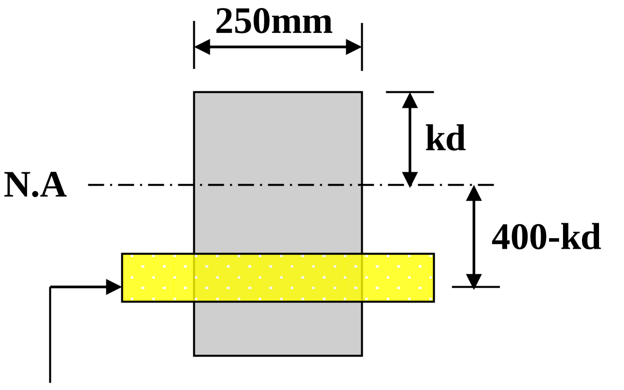
Average stress in concrete = f_c /2

Example : **In a reinforced concrete beam ,b=250mm,d=400mm, A_S =1000mm² and n=8 if the allowable stresses are $f_c \leq$ 12Mpa. and $f_s \leq$ 140Mpa. Determine the maximum bending moment that may be applied.**

Solution :

Computing the factors kd, jd

250*(kd)²/2=8000*(400-kd)



125 (kd)² + 8000kd – 3200000 = 0

(kd)² + 64kd- 25600 = 0

(kd - 131) (kd+195) = 0

Kd=131mm

Jd= 400 - (131/ 3) = 356mm

M_C =1/2* f_c (bkd)(jd)

=1/2*(12*10^6) (0.25*0.131)(0.356) =70KN.m

M_S = f_s A_S (jd)

=140*10^6*1000*10^{-6}*0.356=49.8 KN.m

Maximum bending moment=49.8 KN.m