Composition and Resolution of Force :

Let the force (F) shown in fig.(1) with the direction (θ) We can resolve this force into two components :

1- Horizontal component (Fx) which lies on x- axis

2- Vertical component (Fy) which lies on y- axis as shown in fig.(2)

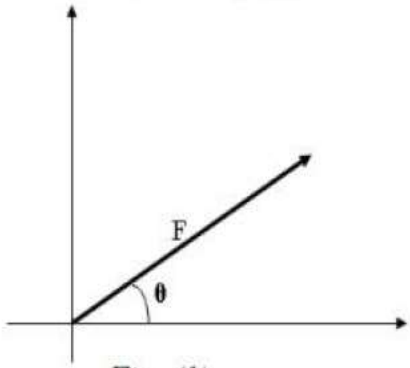


Fig. (1)

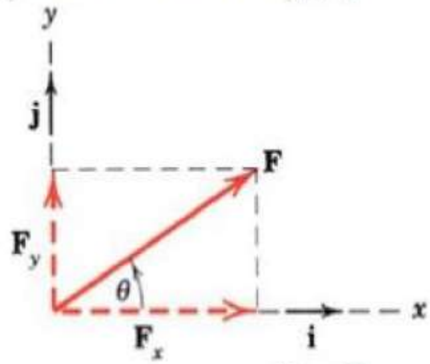


Fig.(2)

Thus from fig.(2) :

The horizontal component may be determined as :

$Fx = F . \cos\theta$

The vertical component may be determined as

$Fy = F . \sin\theta$

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

EX (1) :

Find the two components of the force (100 N) if : θ = 30° , 120° , 270° fig. (2)

Solution :

θ = 30° :

Fx = F . cos θ

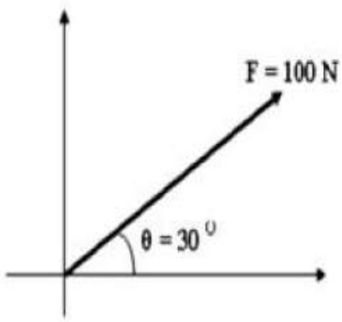
$= 100 * \cos 30$

$= 100 * \frac{\sqrt{3}}{2} = 50\sqrt{3}$ N

Fy = F . sin θ

$= 100 * \sin 30$

$= 100 * 0.5 = 50$ N



θ = 120° :

Fx = F . cos θ

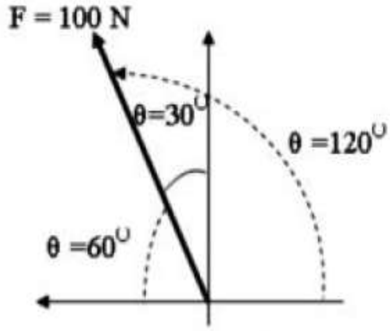
$= 100 * \cos 120$

$= 100 * (-0.5) = -50$ N

Fy = F . sin θ

$= 100 * \sin 120$

$= 100 * \frac{\sqrt{3}}{2} = 50\sqrt{3}$ N



$\theta = 270^\circ$:

$Fx = F . \cos\theta$

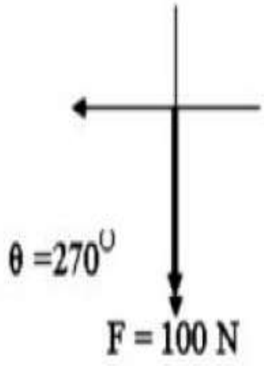
$= 100 * \cos 270$

$= 100 * (0) = 0$

$Fy = F . \sin\theta$

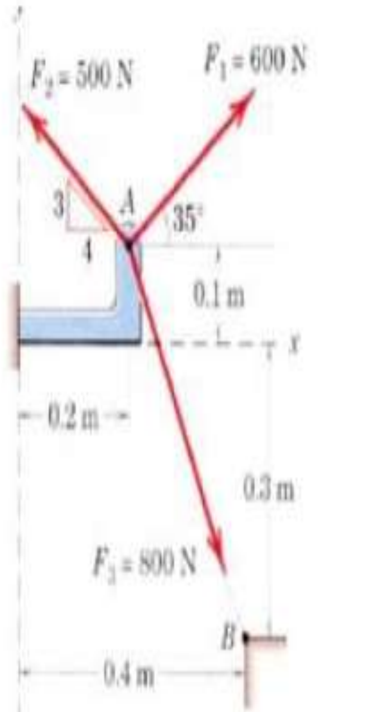
$= 100 * \sin 270$

$= 100 * (- 1) = - 100$ N



EX (2):

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. a, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$

The scalar components of $\mathbf{F}_2$, from Fig. b, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad \textit{Ans.}$$

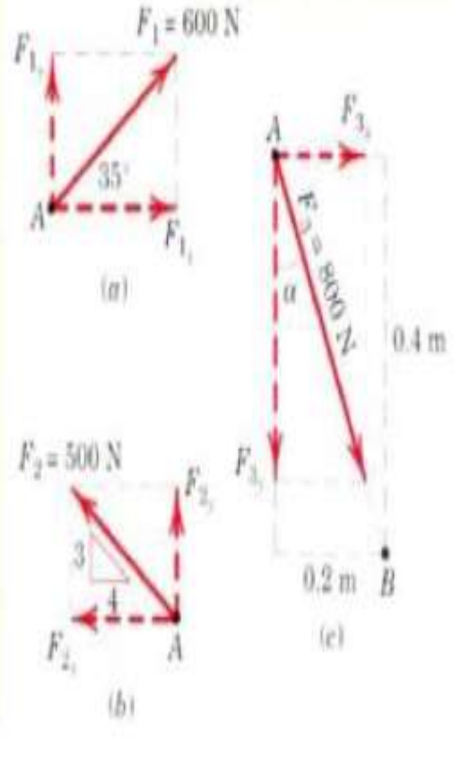
Note that the angle which orients $\mathbf{F}_2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F}_2$ is negative by inspection.

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. c.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

$$\text{Then } F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N} \qquad \textit{Ans.}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N} \qquad \textit{Ans.}$$

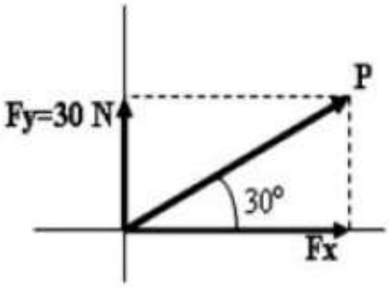


(a) (b) (c)

Ex(3)

The direction of the force (P) is (30^{o}) , Find the horizontal component if the vertical component is (30 N) ?

Solution :



From the diagram shown :

Fy = 30 N

Fy = F . sin θ

30 = P * sin 30

30 = P * 0.5

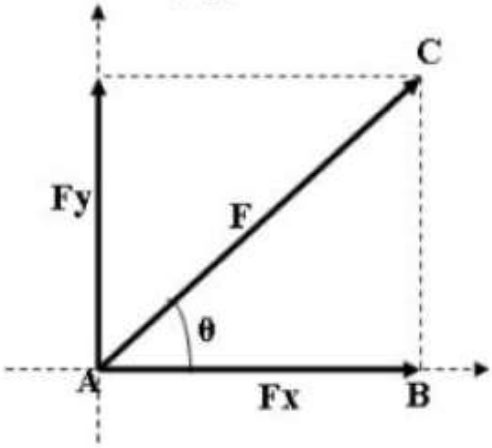
P = 30 / 0.5 = 60 N

Fx = F . cos θ

$$= 60 * \cos 30 = 60 * \frac{\sqrt{3}}{2} = 30\sqrt{3}\ N$$

Composition of Force :

Let we have (Fx) is the horizontal component and (Fy) is the vertical component for the force (F) shown in fig.



From the shape ABC we get :

$$AC^2 = AB^2 + BC^2$$

$$F^2 = Fx^2 + Fy^2$$

$$F = \sqrt{(F_X)^2 + (F_Y)^2}$$

Determination of the direction of force :

The direction of a force can be determined by :

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$$

Ex (4) :

Determine the magnitude and direction of a force (P) , if the horizontal and vertical components are (20 N) , (40 N) respectively ?

Solution :

We have : Fx = 20 N , Fy = 40 N , $F = \sqrt{(F_X)^2 + (F_Y)^2}$

$$F = \sqrt{(20)^2 + (40)^2} = \sqrt{400 + 1600} = \sqrt{20000} = 44.72 \quad N$$

$$\theta = \tan^{-1}(\frac{Fy}{Fx}) = \tan^{-1}(\frac{40}{20}) = 63.43^\circ$$

Resultant of forces system

The resultant is a representative force which has the same effect on the body as the group of forces it replaces.

A simplest force which can replace the original forces system without changing its external effect on a rigid body.

The symbol of resultant force is: ——↠

The unit of resultant force is : Newton (N)

The resultant is applied for different types of forces system as:

1- Coplanar forces system :

a- concurrent coplanar forces system (متلاقية في مستو واحد)

b- non-concurrent coplanar forces system (غير متلاقية في مستو واحد)

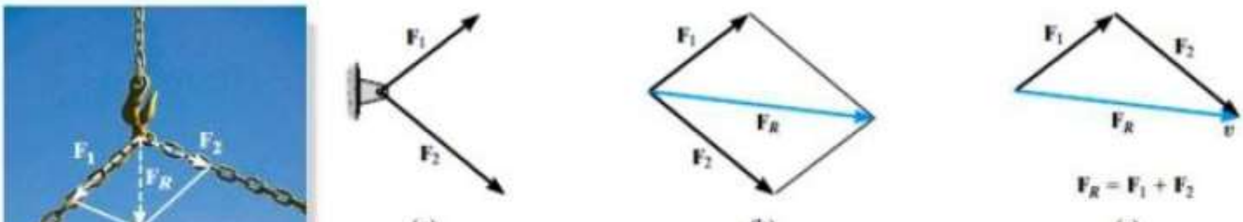
2- Non coplanar forces system :

a- concurrent non-coplanar forces system (متلاقية ليست في مستو واحد)

b- non-concurrent non-coplanar forces system (غير متلاقية ليست فى مستو واحد)

Resultant of concurrent coplanar forces system

Finding a Resultant Force. The two component forces **F1** and **F2** acting on the pin in Fig. *a* can be added together to form the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$, as shown in Fig. *b*. From this construction, or using the triangle rule, Fig. *c*, we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.



(a) (b) (c)

We will find out the resultant force for many forces acting on a rigid body by using the following equations

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3 \mp \ldots \mp F_n.\cos\theta_n$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3 \mp \ldots \mp F_n.\sin\theta_n$$

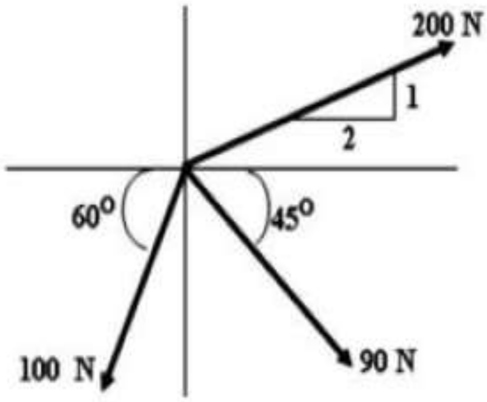
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

The direction of resultant force may be determined as :

$$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right)$$

Ex (5)

Find the resultant force for the concurrent coplanar forces system, shown in figure.



Solution:

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$

$$= 200 \bullet \tfrac{2}{\sqrt{5}} - 100\cos 60 + 90\cos 45 = +192.4N$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$

$$= 200 \bullet \tfrac{1}{\sqrt{5}} - 100\sin 60 - 90\sin 45 = -60.8N$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(192.4)^2 + (60.8)^2} = 202N$$

Ex (6):

Determine the resultant force for the forces system shown in fig.

Solution:

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$
$$= 100\cos 90 + 250\cos(0) - 200\cos 45$$
$$= 192.5 \ \text{N}$$
$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$
$$= 100\sin 90 + 250\sin(0) - 200\sin 45$$
$$= -\ 60.78 \ \text{N}$$
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$
$$= \sqrt{(192.5)^2 + (-60.78)^2} = 201.8 \ \text{N}$$