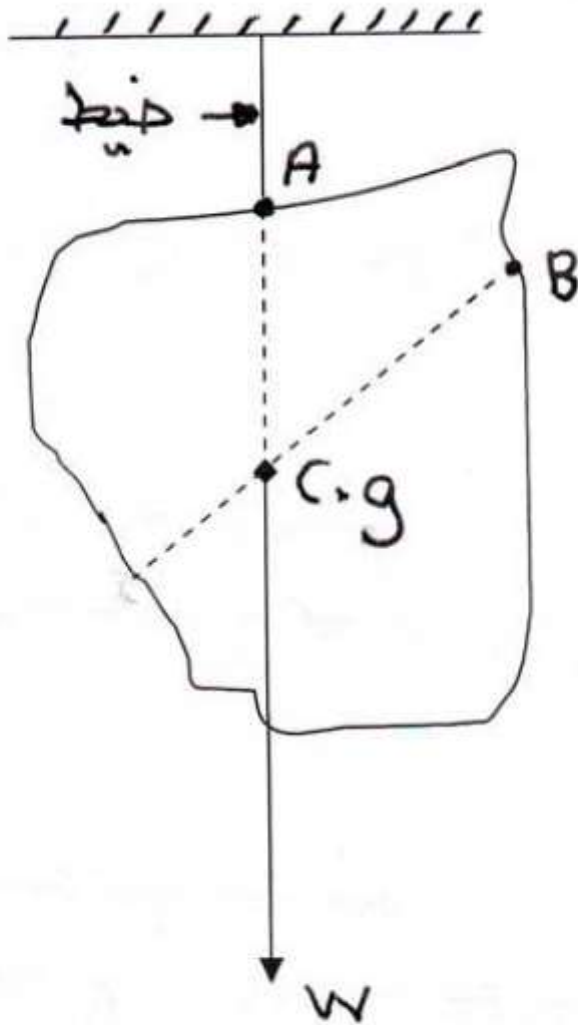


A body of weight (  $W$  ) is supported by string attached at (  $A$  ) as shown in Fig . the only external force acting on the body are its weight and reaction exerted by string . Equilibrium of the body can exist only if their two forces are equal in magnitude opposite in direction and collinear the line of action of the weight (  $W$  ) can be determined by the line of action of support let the body be supported in a new position by the string now attached to (  $B$  ) . the body will shift its position so that the line of action of the weight is again collinear with the string .

These two position of the line of action of the weight are determined experimentally . the intersection of these two lines represents the center of gravity (  $c . g$  ) where the weight passes through it .



### center of gravity of simple area and length

مركز ثقل المساحة البسيطة والطول

We can find the laws of center of gravity of the simple area from the formula

$$\bar{X} = \frac{\int x dA}{A} \quad \bar{Y} = \frac{\int y dA}{A}$$

معادلات استخراج مراكز ثقل المساحات البسيطة

Also  $\therefore$  we can find the center of gravity of the length from these formula المعادلات

$$\bar{X} = \frac{\int X dL}{L} \qquad \bar{y} = \frac{\int y dL}{L}$$

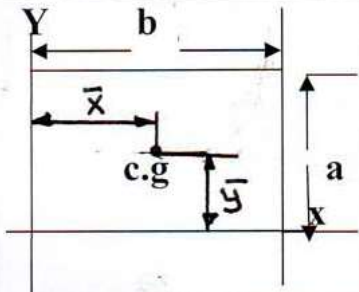
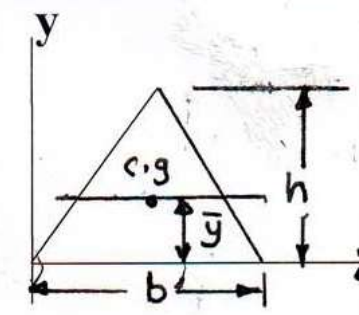
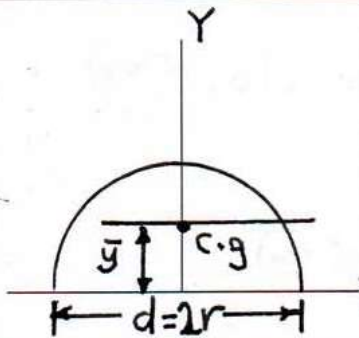
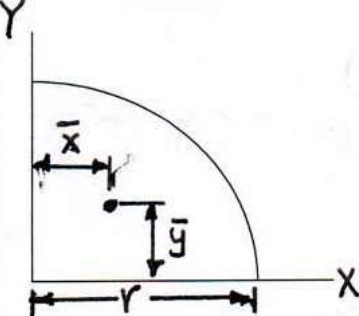
$\bar{X}$  ,  $\bar{y}$  = coordinate of center of gravity احداثيات مركز الثقل  
A = simple area المساحة البسيطة

L = Length الطول

dA = differentiation of area مشتقة المساحة

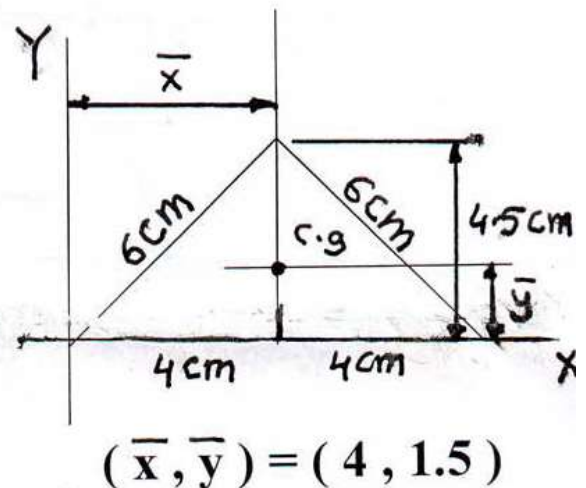
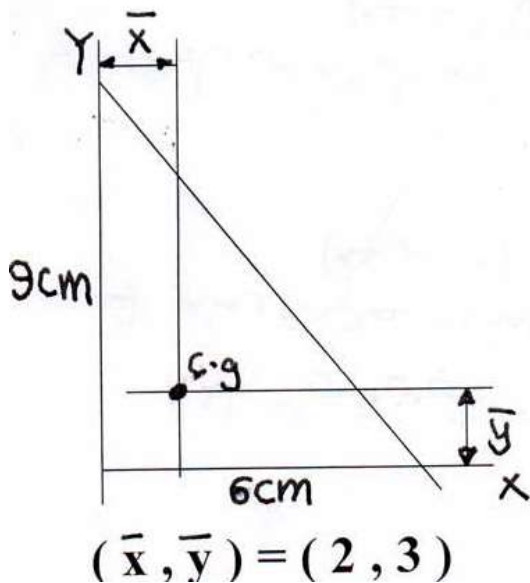
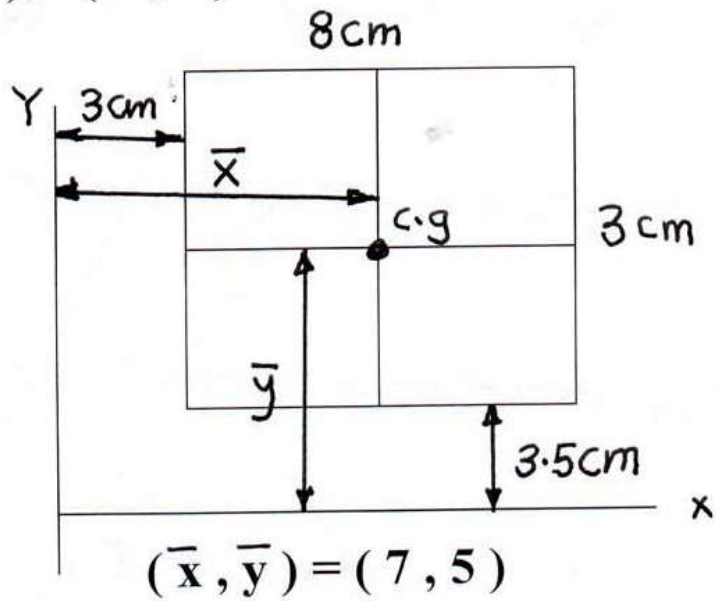
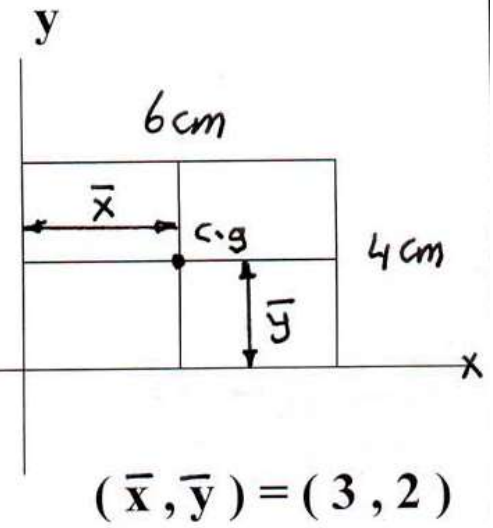
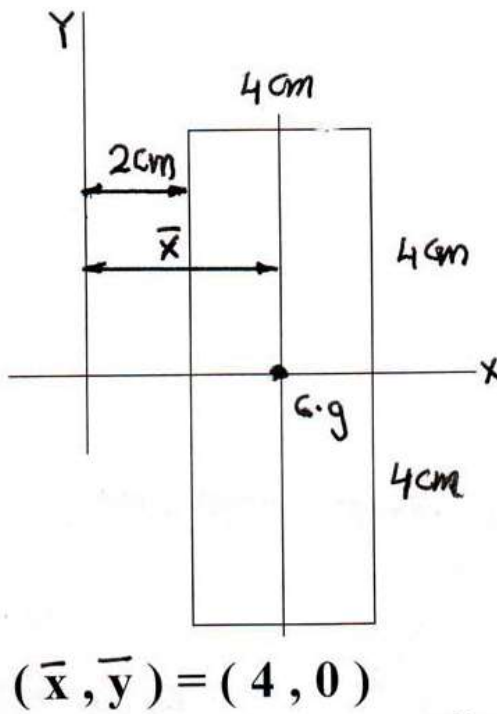
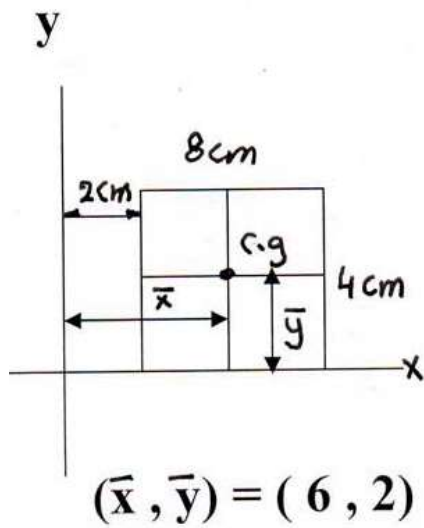
dL = = = length مشتقة الطول

The table shown below shows the laws to find the center of gravity of simple area and length.

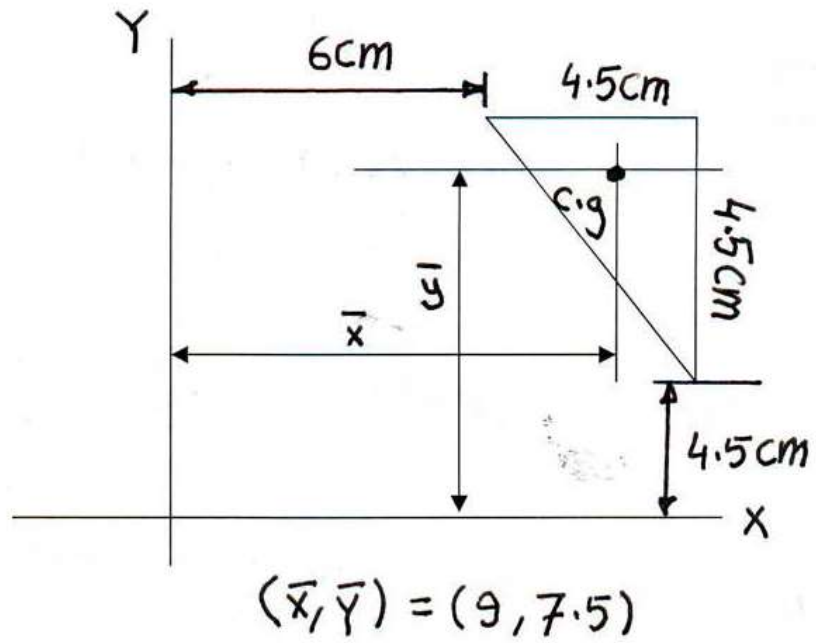
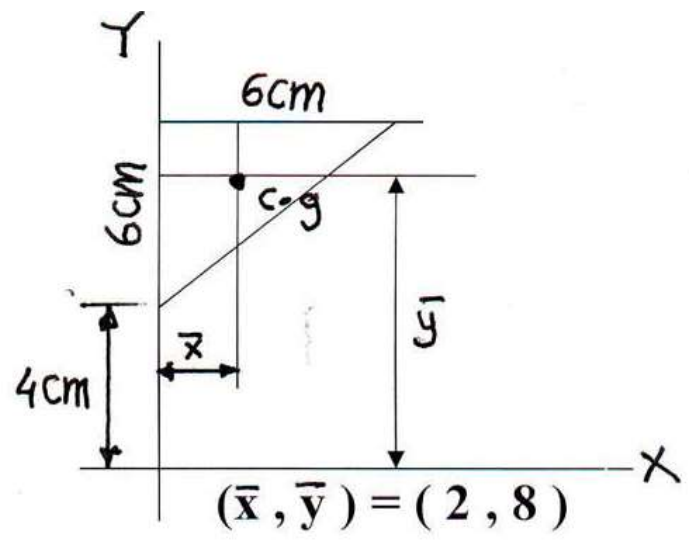
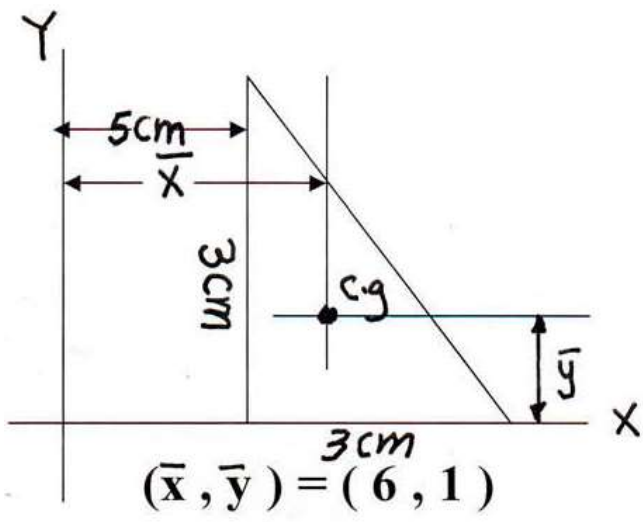
| Shape الشكل  | Area or length<br>المساحة او الطول | $\bar{X}$                  | $\bar{y}$                  |
|--|------------------------------------|----------------------------|----------------------------|
|    | $ab$                               | $\frac{b}{2}$              | $\frac{a}{2}$              |
|   | $\frac{bh}{2}$                     | ---                        | $\frac{h}{3}$              |
|  | $\frac{\pi r^2}{2}$                | 0                          | $\frac{4r}{3\pi} = 0.424r$ |
|  | $\frac{\pi r^2}{4}$                | $\frac{4r}{3\pi} = 0.424r$ | $\frac{4r}{3\pi} = 0.424r$ |

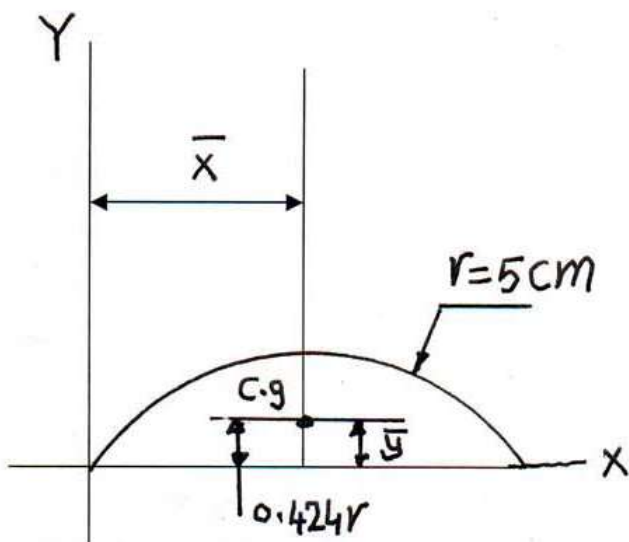
Q/Find the center of gravity of simple area shown in Fig .

اوجد مركز ثقل المساحة البسيطة الموضحة في الشكل



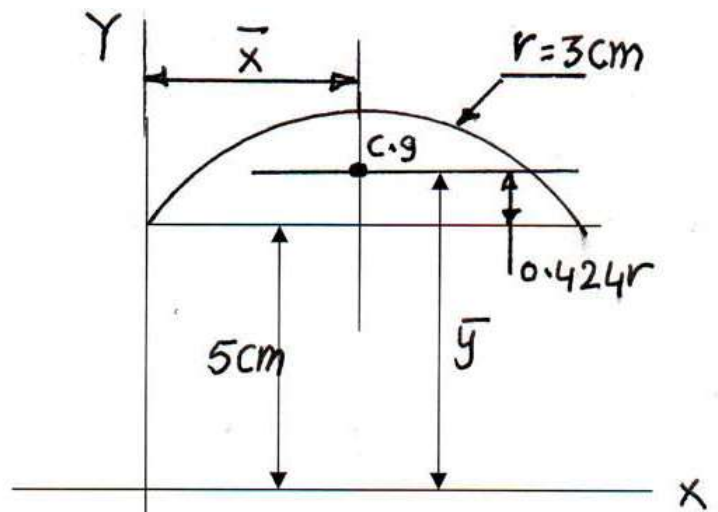






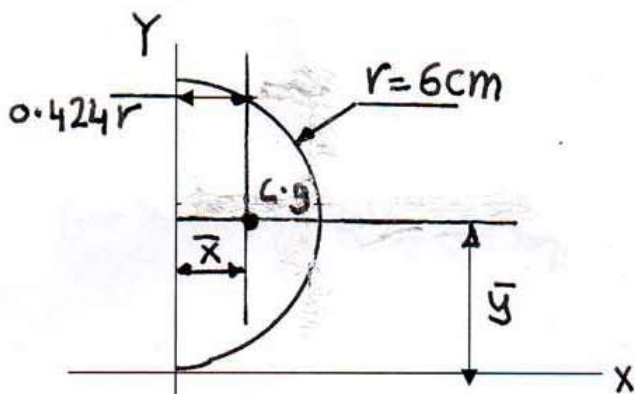
$$\begin{aligned}\bar{X} &= 5 \text{ cm} \\ \bar{Y} &= 0.424 \cdot 5 \\ \bar{Y} &= 2.12 \text{ cm}\end{aligned}$$

$$(\bar{x}, \bar{y}) = (5, 2.12)$$



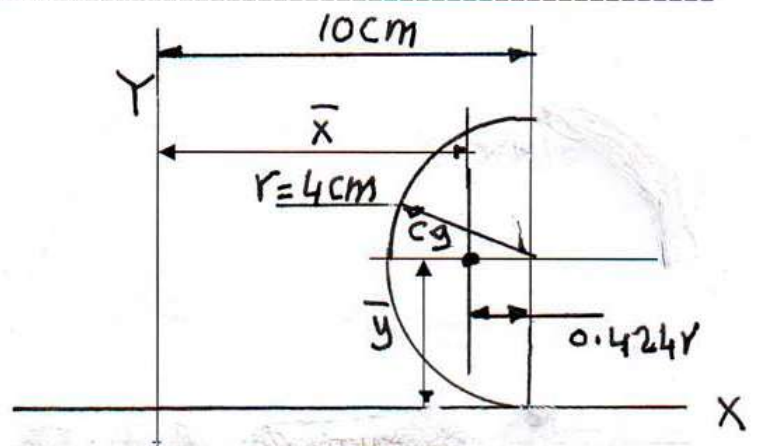
$$\begin{aligned}\bar{x} &= 3 \text{ cm} \\ \bar{y} &= 5 + 0.424 \cdot 3 \\ \bar{y} &= 6.272 \text{ cm}\end{aligned}$$

$$(\bar{x}, \bar{y}) = (3, 6.272)$$



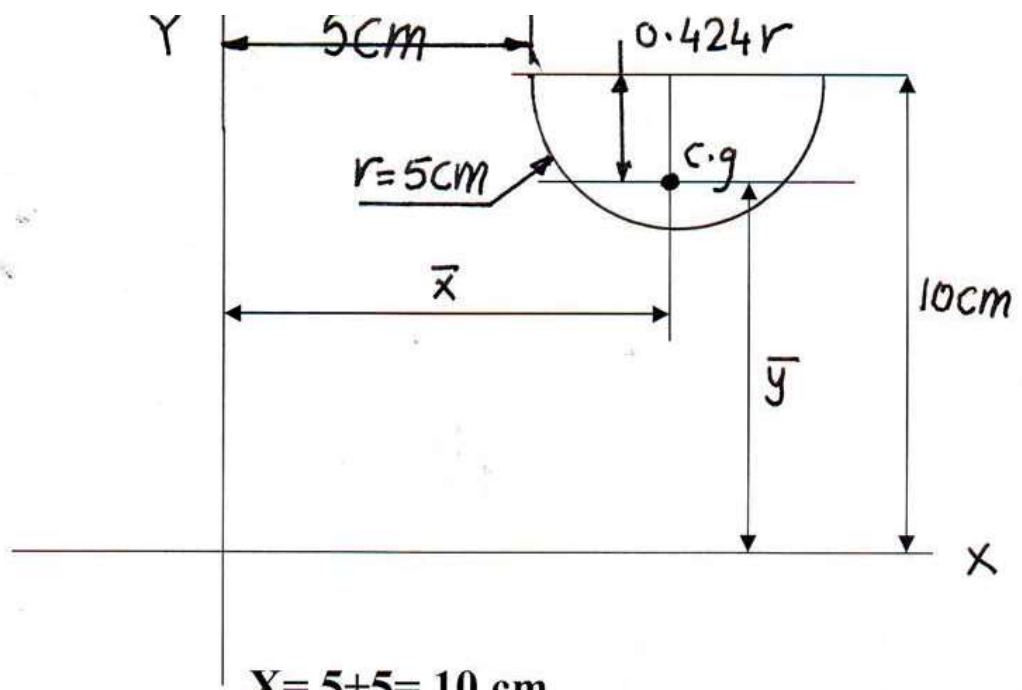
$$\begin{aligned}\bar{X} &= 0.424 \cdot 6 \\ \bar{X} &= 2.544 \text{ cm} \\ \bar{Y} &= 6 \text{ cm}\end{aligned}$$

$$(\bar{x}, \bar{y}) = (2.544, 6)$$

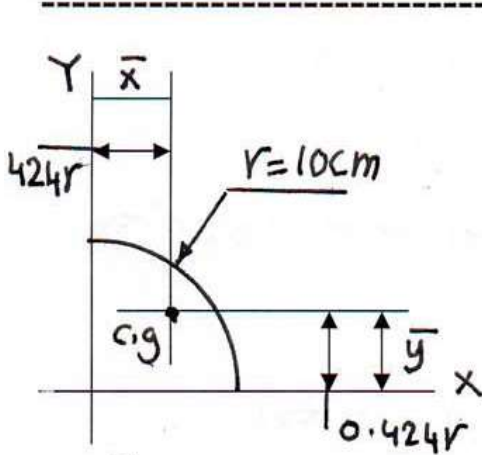


$$\begin{aligned}\bar{x} &= 10 - 0.424 \cdot 4 \\ \bar{x} &= 8.304 \text{ cm} \\ \bar{y} &= 4 \text{ cm}\end{aligned}$$

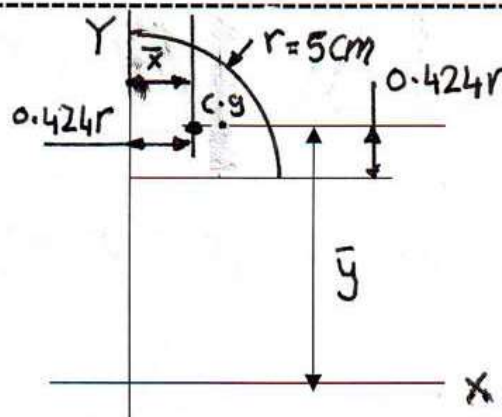
$$(x, y) = (8.304, 4)$$



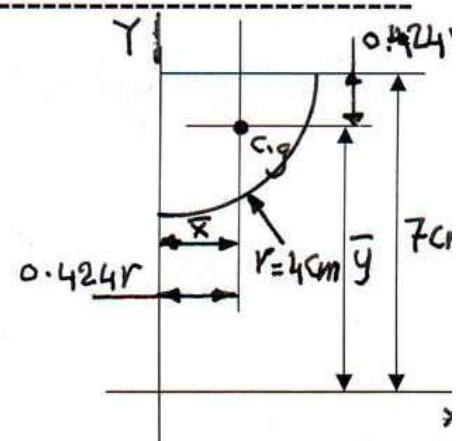
$$\begin{aligned}
 X &= 5 + 5 = 10\text{ cm} \\
 Y &= 10 - 0.424 \cdot 5 \\
 &= 10 - 2.12 \\
 &= 7.88\text{ cm} \\
 (x, y) &= (10, 7.88)
 \end{aligned}$$



$$\begin{aligned}
 \bar{x} &= 0.424 \cdot 10 \\
 \bar{x} &= 4.24\text{ cm} \\
 \bar{y} &= 0.424 \cdot 10 \\
 \bar{y} &= 4.24\text{ cm} \\
 (\bar{x}, \bar{y}) &= (4.24, 4.24)
 \end{aligned}$$

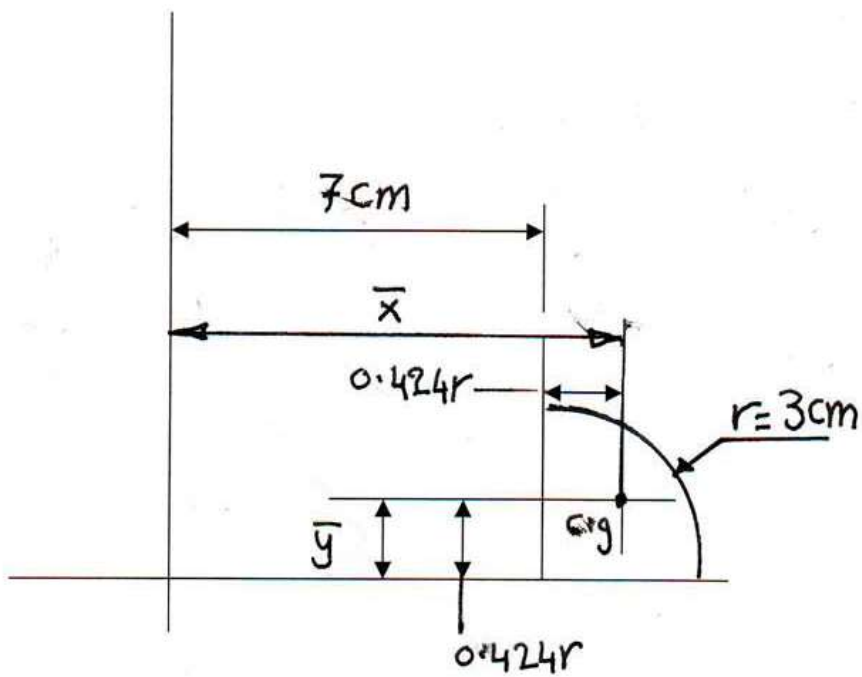


$$\begin{aligned}
 \bar{x} &= 0.424 \cdot 5 \\
 \bar{x} &= 2.12\text{ cm} \\
 \bar{y} &= 5 + 0.424 \cdot 5 \\
 \bar{y} &= 5 + 2.12 \\
 \bar{y} &= 7.12\text{ cm} \\
 (\bar{x}, \bar{y}) &= (2.12, 7.12)
 \end{aligned}$$

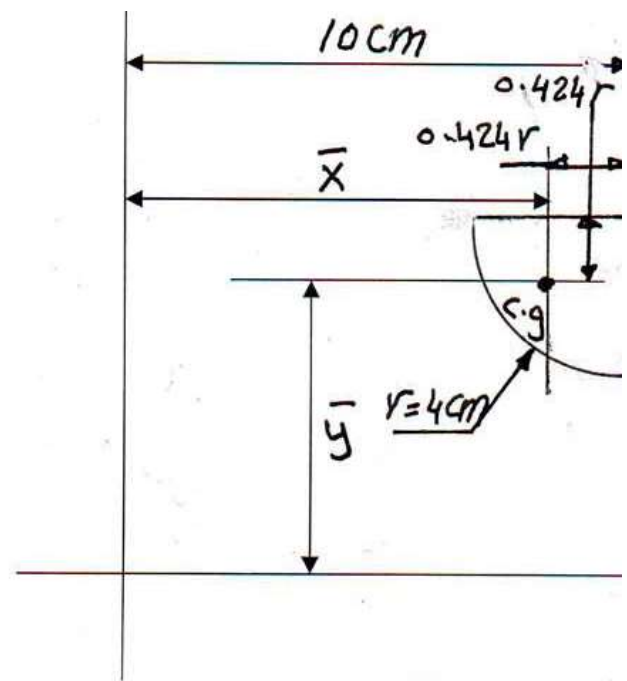


$$\begin{aligned}
 \bar{x} &= 0.424 \cdot 4 \\
 \bar{x} &= 1.696\text{ cm} \\
 \bar{y} &= 7 - 0.424 \cdot 4 \\
 \bar{y} &= 7 - 1.696 \\
 \bar{y} &= 5.304\text{ cm} \\
 (\bar{x}, \bar{y}) &= (1.696, 5.304)
 \end{aligned}$$





$$\begin{aligned}\bar{x} &= 7 + 0.424 \cdot 3 \\ &= 7 + 1.272 \\ &= 8.272 \text{ cm} \\ \bar{y} &= 0.424 \cdot 3 \\ \bar{y} &= 1.272 \text{ cm} \\ (\bar{x}, \bar{y}) &= (8.272, 1.272)\end{aligned}$$



$$\begin{aligned}\bar{x} &= 10 - 0.424 \cdot 4 \\ &= 10 - 1.696 \\ &= 8.304 \text{ cm} \\ \bar{y} &= 9 - 0.424 \cdot 4 \\ \bar{y} &= 9 - 1.696 \\ &= 7.304 \text{ cm} \\ (\bar{x}, \bar{y}) &= (8.304, 7.304)\end{aligned}$$