Lecture (1,2)

Types of unit systems, density, specific volume, and pressure, temperature (Celsius and absolute), Properties of fluids: difference between fluids and solid metals, difference between liquids and gases. Definition of density, relative density, specific weight, specific volume, ideal fluid, real fluid, examples.

System of units

As any quantity can be expressed in whatever way you like it is sometimes easy to become confused as to what exactly or how much is being referred to. This is particularly true in the field of fluid mechanics. Over the years many different ways have been used to express the various quantities involved. Even today different countries use different terminology as well as different units for the same thing - they even use the same name for different things e.g. an American pint is 4/5 of a British pint! To avoid any confusion on this course we will always use the SI (metric) system

The SI System of units

The SI system consists of six **primary** units, from which all quantities may be described. For convenience **secondary** units are used in general practices which are made from combinations of these primary units.

Primary Units

The six **primary** units of the SI system are shown in the table below:

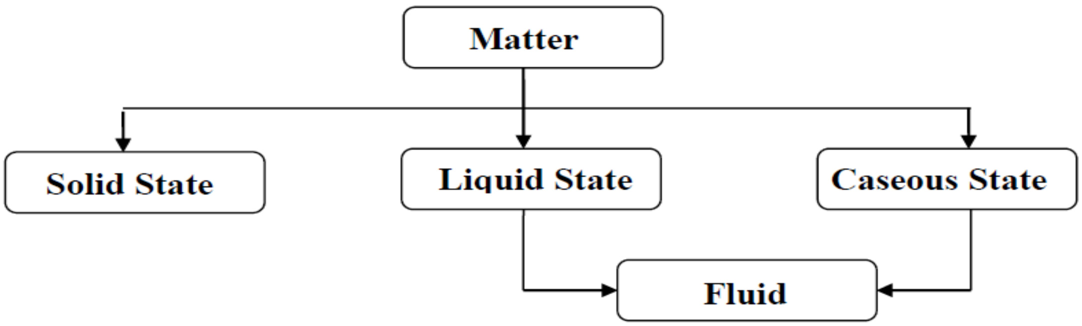
Quantity	SI Unit	Dimension
length	meter, m	L
mass	kilogram, kg	M
time	second, sec	T
temperature	F, K,C,R	θ
current	ampere, *A*	I
luminosity	candela	Cd

In fluid mechanics we are generally only interested in the top four units from this table. Notice how the term 'Dimension _of a unit has been introduced in this table. This is not a property of the individual units; rather it tells what the unit represents. For example, a meter is a length which has a Dimension L but also, an inch, a mile or a kilometer is all lengths so have dimension of L. (The above notation uses the MLT system of dimensions, there are other ways of writing dimensions – we will see more about this in the section of the course on dimensional analysis.)

Quantity measured	***laws***	***Basic units***	***symbol***
Volume (v)		Liter or cubic-meter	L or m^3
Force (F)	Force= m * g	Newton ($\frac{kgm}{sec^2}$)	N
Energy (work)	Work= F *distance	Joule (N * m)	J
power	Power = work / time	Watt (J/sec)	Watt
Pressure	Pressure=force/area	Bar or pascal=N/m^3	Bar or pa
Ground acceleration (g)		m/sec^2	m/sec^2
Specific volume (υ)	υ = volume/mass	m^3/kg	m^3/kg
Mass Density (ρ)	ρ =mass/volume	Kg/m^3	
Dynamic viscosity (μ)		Pa.sec	(N*sec)/m^2

Fluids Mechanics and Fluid Properties

What is fluid mechanics? As its name suggests it is the branch of applied mechanics concerned with the statics and dynamics of fluids - both liquids and gases. The analysis of the behavior of fluids is based on the fundamental laws of mechanics which relate continuity of mass and energy with force and momentum together with the familiar solid mechanics properties.



Basic difference: molecules distance; activities; structures
المسافة بين الجزيئات , نشاط الجزيئات , بنية الجزيئات

Properties of Fluids

1- Density

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

2- Mass Density

Mass Density, ρ , is defined as the mass of substance per unit volume.

Units: Kilograms per cubic meter, (or kg/m3)

3- Specific Weight or specific gravity

Specific Weight **γ** , (sometimes ω, and sometimes known as ***specific gravity***) is defined as the weight per unit volume. Or the force exerted by gravity, g, upon a unit volume of the substance. The Relationship between g and ω can be determined by Newton 's 2nd Law, since **Weight per unit volume = mass per unit volume** × **g**

$\gamma = \rho * g$

Newton 's per cubic meter, N/m3

Example:4// The specific weight of water at ordinary pressure and temperature is 9.81KN/m^3. The specific gravity of mercury is 13.55. Compute the density of water and the specific weight and density of mercury.

Solution:

$$\rho_{water} = \frac{\gamma_{water}}{g} = \frac{9.81\,{}^{KN}/{}_{m^3}}{9.81\,{}^{m}/{}_{s^2}} = 1.00\,{}^{Mg}/{}_{m^3} = 1000\,{}^{Kg}/{}_{m^3} = 1.00\,{}^{g}/{}_{cm^3} \qquad Ans.$$

$$\gamma_{mercury} = s.g._{mercury} \times \gamma_{water} = 13.55(9.81) = 133\,{}^{KN}/{}_{m^3} \qquad Ans.$$

$$\rho_{mercury} = s.g._{mercury} \times \rho_{water} = 13.55(1.00) = 13.55\,{}^{Mg}/{}_{m^3} \qquad Ans.$$

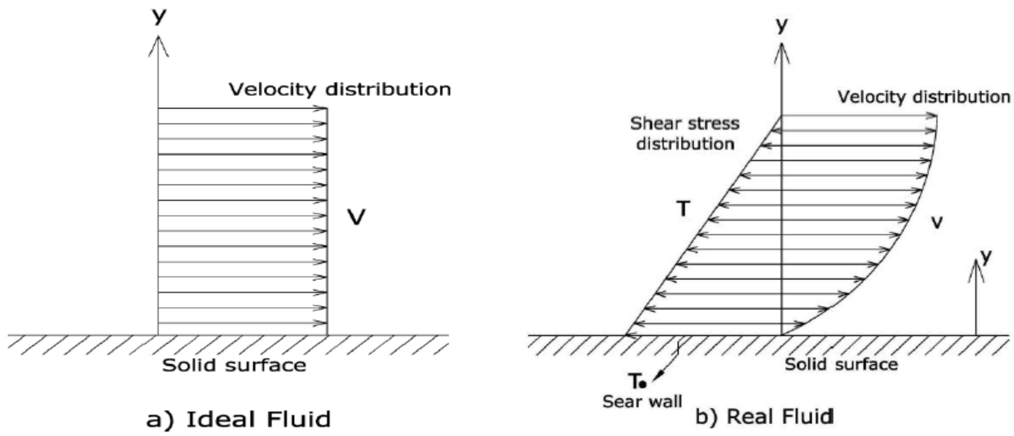
4- Relative Density

Relative Density R.D is defined as the ratio of mass density of a substance to some standard mass density. For solids and liquids this standard mass density is the maximum mass density for water at atmospheric pressure.

$$R.D = \frac{\rho_{sub}}{\rho_{ref}}$$

5- Viscosity

Viscosity, μ **,** is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity such as syrup deforms more slowly than fluid with a low viscosity such as water. All fluids are viscous; ―*Newtonian Fluids*‖ obey the linear relationship given by Newton 's law of viscosity. Which we were saw earlier. Where τ is the *shear stress*



Example: Two parallel plates of distance 1.25cm filled by oil with viscosity μ=14poises. Calculate the shear stress if du=2.5m/s.

Solution:

dh=1.25/100=0.0125m

μ=14/10=1.4 pa.s

$\tau=\mu\frac{du}{dh}$ = 1.4 x 2.5 /0.0125=280 pa.

6- Coefficient of Dynamic Viscosity

The ***Coefficient of Dynamic Viscosity,*** μ **,** is defined as the shear force, per unit area, required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{F}{A}}{\frac{du}{dy}} = \frac{F}{A} * time = \frac{mass}{length * area}$$

7- Kinematic Viscosity

Kinematic Viscosity, ν **,** is defined as the ratio of dynamic viscosity to mass density.

$$\nu = \frac{\mu}{\rho}$$

Problem 1: Dynamic and Kinematic Viscosity

A liquid has a **density** of 900 kg/m^{3}and a **dynamic viscosity** of 0.45 Pa\cdotps.
Find its **kinematic viscosity**.

$$\nu = \frac{\mu}{\rho}$$

$= \frac{\mathbf{0.54}}{\mathbf{900}}$=**0.0005**

Example : Two large parallel plates are separated by a distance of 5 mm. The lower plate is stationary, and the upper plate moves at a constant velocity of 0.2 m/s. If the fluid between the plates has a **dynamic viscosity** of 0.1 Pa\cdotps, find the **shear stress** in the fluid.

Use the Couette flow shear relation $\tau = \mu \frac{du}{dy}$.
For two parallel plates with the upper plate speed $U$and gap h, $du/dy = U/h$.

Given:

- $U = 0.2$ m/s

- $h = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$
- $\mu = 0.1 \text{ Pa} \cdot \text{s}$

Compute:

$$\tau = 0.1 \cdot \frac{0.2}{5 \times 10^{-3}} = 0.1 \cdot 40 = 4 \text{ Pa.}$$

So, the shear stress in the fluid is **4 Pa** (directed opposite the plate motion). This assumes a Newtonian fluid and steady, linear (laminar) velocity profile.

8- Pressure

As mentioned above a fluid will exert a normal force on any boundary it is in contact with. Since these boundaries may be large and the force may differ from place to place it is convenient to work in terms of pressure, *p*, which is the force per unit area. If the force exerted on each unit area of a boundary is the same, the pressure is said to be *uniform*.

$$p = \frac{F}{A} = \frac{N}{m^2}$$

9- Temperature

Liquids vs. Gasses

Although liquids and gasses behave in much the same way and share many similar characteristics, they also possess distinct characteristics of their own. Specifically

• A liquid is difficult to compress and often regarded as being incompressible.

A gas is easily to compress and usually treated as such - it changes volume with pressure.

• A given mass of liquid occupies a given volume and will occupy the container it is in and form a free surface (if the container is of a larger volume).A gas has no fixed volume, it changes volume to expand to fill the containing vessel. It will completely fill the vessel so no free surface is formed.

Fluids vs. (Versus) Solids

In the above we have discussed the differences between the behavior of solids and fluids under an applied force. Summarizing, we have;

1. For a **solid** the strain is a function of the applied stress (providing that the elastic limit has not been reached). For a **fluid**, the rate of strain is proportional to the applied stress.

2. The strain in a **solid** is independent of the time over which the force is applied and (if the elastic limit is not reached) the deformation disappears when the force is removed. A **fluid** continues to flow for as long as the force is applied and will not recover its original form when the force is removed. It is usually quite simple to classify substances as either solid or liquid. Some substances, however, (glass) appear solid under their own weight. Pitch will, although appearing solid at room temperature, deform and spread out over days - rather than the fraction of a second it would take water. As you will have seen when looking at properties of solids, when the elastic limit is reached, they seem to

flow. They become plastic. They still do **not** meet the definition of true fluids as they will only flow after a certain minimum shear stress is attained.

Comparison Chart

Solid	*Liquid*	*Gas*
Fixed shape and volume	Takes the shape of the container	Takes the shape and volume of its container
Harder to compress	Hard to compress	Compressible
Does not flow easily	Flows easily	Flows easily
Ice (cooled water)	Water	Steam (heated water)Source:

Newtonian &Non-Newtonian Fluids

Even among fluids which are accepted as fluids there can be wide differences in behavior under stress. Fluids obeying Newton 's law where the value of constant are known as **Newtonian** fluids. If constant the shear stress is linearly dependent on velocity gradient. This is true for most common fluids. Fluids in which the value of not constant are known as ***non-Newtonian*** fluids. There are several categories of these, and they are outlined briefly below. These categories are based on the relationship between shear stress and the velocity gradient (rate of shear strain) in the fluid. These relationships can be seen in the graph below for several categories

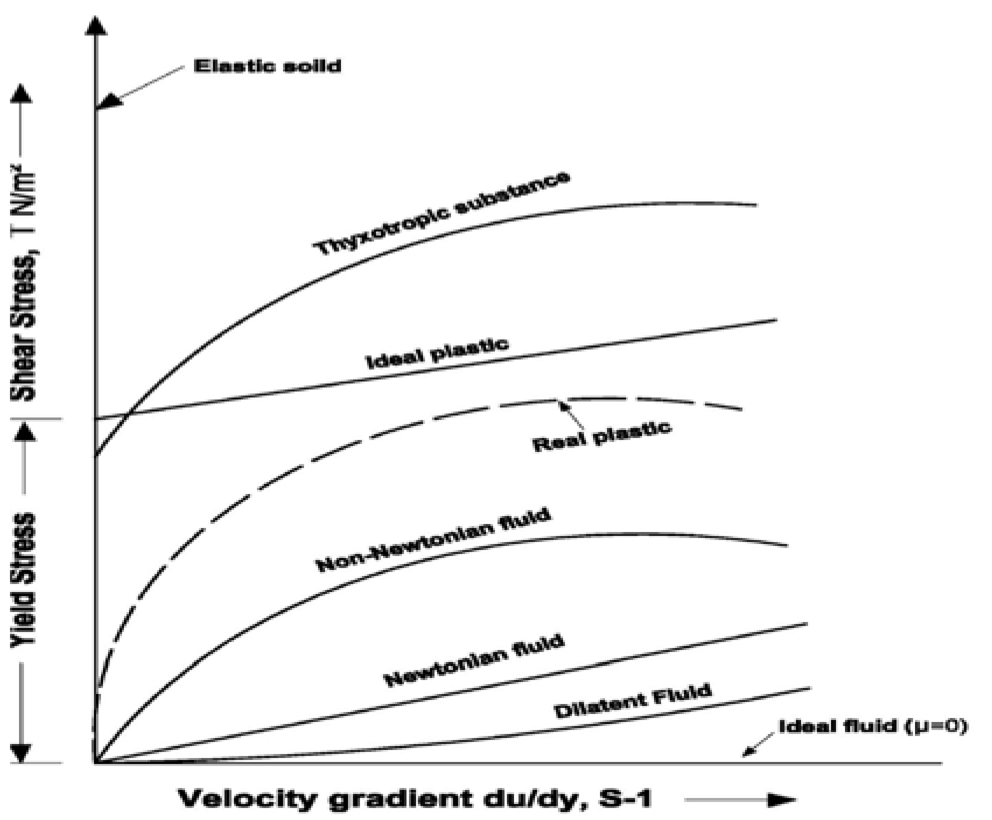


Diagram of types of fluid according to the viscosity

Pressure(P):- **It is the force distributed over an area, or the force per unit area. It exists wherever fluid exist, either at rest or in motion.**

$$P = \frac{F(force)}{A(area)}$$

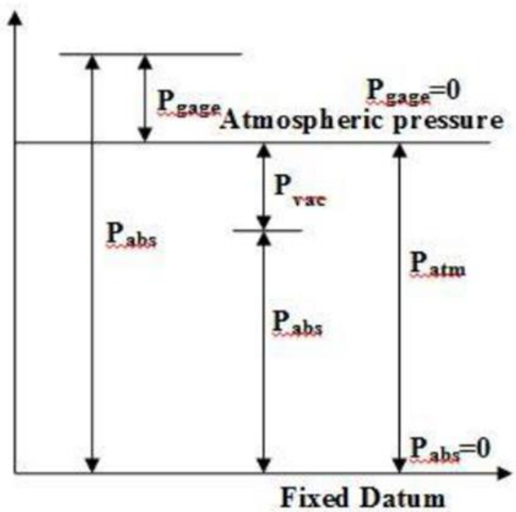
The dimensions are : $M/_{LT^2}$

The units of pressure are : $N/_{m^2} = \boldsymbol{Pa}$

The standard atmospheric pressure = 101.3 KPa = 1Bar.

Absolute, vacuum and gage pressure

- **Atmospheric pressure P_{atm}:** the pressure that applied on the surfaces in contact with air. It is also known as "Barometric pressure". The atmospheric pressure at sea level (above absolute zero) is called "Standard atmospheric pressure".
- **Gage pressure (P_{gage}):** it is the pressure, measured with help of pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.



- **Vacuum pressure (P_{vac})**: is the different between the absolute and the atmospheric pressure when the first is lower than the second. Or it is the negative pressure relative to the gage pressure.
- **Absolute pressure (P_{abs}):** is the pressure relative to the absolute vacuum pressure.

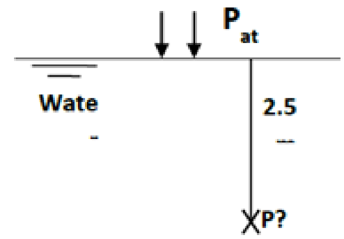
$$P_{abs} = P_{atm} + P_{gage} \qquad P_{abs} = P_{atm} - P_{vac}$$

Where P_{gage} may be positive or negative (vacuum, suction).

Absolute, vacuum and gage pressure

Example 9// Determine the pressure at a point which is 2.5m below the surface of clear water. Express it as N/m2 both in absolute and gage units. Assume P_{atm}= 101.325 kpa

Solution:-



$P = \gamma h$

$P_{gage} = \frac{9810}{1000} \times 2.5 = 24.525 kpa$ ....Ans.

$P_{abs} = P_{gage} + P_{atm} = 24.525 + 101.325 = 125.85 Kpa$ . ....Ans.

Example: A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

Pvac = Patm _ Pabs

5.8=14.5-Pabs

Pabs=14.5-5.8=8.7psi

Temperature Scales

Kalvin scale K= C +273 -

Rankine scale T(R)= F + 460 -

T(R)=1.87 (K)

T(F)=1.87C +32

EX:1// Convert the 65°C to °K, °F ,and °R.

T(K)= 65 + 273=338

T(F)=(1.8 x 65) + 32=153.55

T(R)=(1.87 x338)=632

Ex2: Convert 760°R to °C, °F and °K.

T(R)=1.87(K)

760=1.87 x K

K= 760 /1.87 = 406

BUT: K=C +273 406=C +273

C=133

T(F)=1.87C +32 T(F)= (1.87 x 133) +32=280.7

EX3*:* Expressing Temperature Rise in Different Units

During a heating process, the temperature of a system rises by 10°C. Express this rise in temperature in K, °F, and R. (H.W).

Ideal fluid an ideal fluid is a fluid that has several properties including the fact that it is: 1- Incompressible – the density is constant

2- I rotational – the flow is smooth, no turbulence

3- Non viscous fluid has no internal friction ($\eta = 0$)

Real fluid Fluid that has viscosity ($\mu > 0$) and their motion known as viscous flow, all the fluids in actual practice are real fluids. Fluid dynamics: We use ideal fluid:

1. Continuity equation
2. Bernoulli equation
3. Newtonian Fluids: A real fluid in which the shear stress is directly proportional to rate of shear strain (or velocity gradient).
4. Non-Newtonian Fluid: A real fluid in which the shear stress is not proportional to the rate of shear strain.
5. Ideal Plastic Fluid:

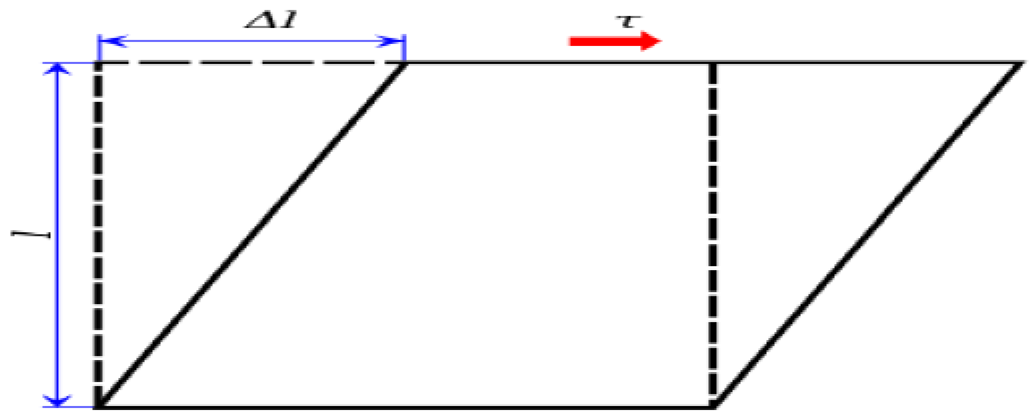
A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient)

Lecture 3

In physics, fluid dynamics is a sub discipline of fluid mechanics that deals with fluid flow—the science of fluids (liquids and gases) in motion. It has several sub disciplines itself, including aerodynamics (the study of air and other gases in motion) and hydrodynamics (the study of liquids in motion). Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines, predicting weather patterns, understanding nebulae in interstellar space and modeling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid, and crowd dynamics.

Fluid dynamics offers a systematic structure—which underlies these practical disciplines—that embraces empirical and semi-empirical laws derived from flow measurement and used to solve practical problems. The solution to a fluid dynamics problem typically involves calculating various properties of the fluid, such as flow velocity, pressure, density, and temperature, as functions of space and time. Before the twentieth century, hydrodynamics was synonymous with fluid dynamics. This is still reflected in names of some fluid dynamics topics, like magneto hydrodynamics and hydrodynamic stability, both of which can also be applied to gases.

A shear stress is defined as the component of stress coplanar with a material cross section. Shear stress arises from the force vector component parallel to the cross section. Normal stress, on the other hand, arises from the force vector component perpendicular to the material cross section on which it acts.



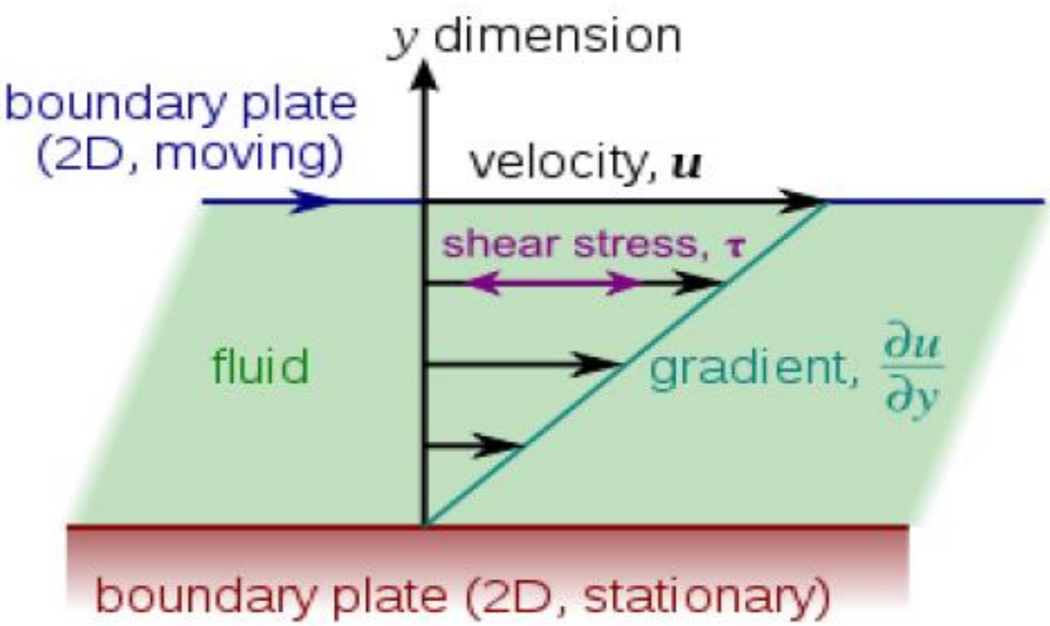
The formula to calculate average shear stress is force per unit area.

$$\tau = \frac{F}{A}$$

Where: τ = the shear stress; F = the force applied; A = the cross-sectional area of material with area parallel to the applied force vector. ***Shear stress in fluids*** Any real fluids (liquids and gases included) moving along solid boundary will incur a shear stress on that boundary. The no-slip condition dictates that the speed of the fluid at the boundary (relative to the boundary) is zero, but at some height from the boundary the flow speed must equal that of the fluid.

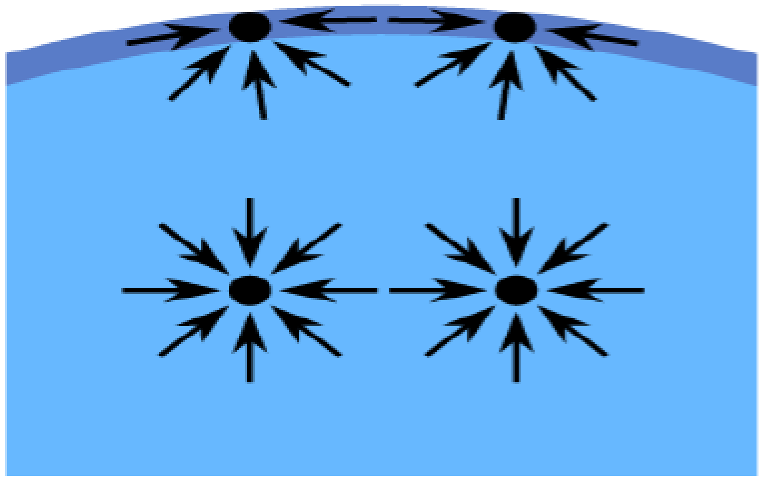
The region between these two points is aptly named the boundary layer. For all Newtonian fluids in laminar flow the shear stress is proportional to the strain rate in the fluid where the viscosity is the constant of proportionality. However, for non-Newtonian fluids, this is no longer the case as for these fluids the viscosity is not constant. The shear stress is imparted onto the boundary as a result of this loss of velocity. The shear stress, for a Newtonian fluid, at a surface element parallel to a flat plate, at the point y, is given by:

$$d\tau = \mu * \frac{du}{dy} \rightarrow \tau = \mu \frac{u}{y}$$



Surface Tension Surface tension is a contractive tendency of the surface of a liquid that allows it to resist an external force. It is shown, for example, in the floating of some objects on the surface of water, even though they are denser than water, and in the ability of some insects (e.g., water striders) to run on water's surface. This property is caused by cohesion of similar molecules and is responsible for many of the behaviors of liquids. The cohesive forces among liquid molecules are responsible for the phenomenon of surface tension, as shown in figure.

In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the surface do not have other molecules on all sides of them and therefore are pulled inwards. This creates some internal pressure and forces liquid surfaces to contract to the minimal area.



Capillary Action **Definition** Capillary action is the **rise or fall of a liquid** in a narrow tube (capillary) or in small pores and spaces, **due to surface tension, adhesive forces, and cohesive forces.**

It happens **without any external force** like a pump — it's caused by molecular interactions between the liquid and the solid surface.

1. Forces involved

There are two main forces:

1. **Cohesive force** — attraction between molecules of the **same** liquid.
   Example: Water molecules attract each other.
2. **Adhesive force** — attraction between molecules of the **liquid and the solid** surface.
   Example: Water molecules attracted to glass molecules.

2. How it works

When a narrow glass tube is dipped in a liquid:

- If **adhesive force > cohesive force** (like water in glass):
  → the liquid climbs up the tube — **capillary rise**.

- If **cohesive force > adhesive force** (like mercury in glass):
  → the liquid level goes down — **capillary depression**.

3. Shape of the meniscus

- **Concave meniscus** → liquid rises (e.g., water in glass).
- **Convex meniscus** → liquid falls (e.g., mercury in glass).

4. Formula for capillary rise or fall

The height h of the rise (or fall) in a tube of radius r is given by:

$$\boxed{h = \frac{2T\cos\,\theta}{\rho g r}}$$

where:

- T= surface tension of the liquid (N/m)
- θ= angle of contact between liquid and tube
- ρ= density of the liquid (kg/m³)
- g= acceleration due to gravity (m/s²)
- r= radius of the capillary tube (m)

5. Explanation of the formula

- Smaller tube radius → larger rise.
- Larger surface tension → larger rise.
- If the contact angle $\theta < 90°$(like water), $\cos\theta$is positive → **liquid rises**.
- If $\theta > 90°$(like mercury), $\cos\theta$is negative → **liquid falls**.

6. Examples of Capillary Action

1. **Water in a thin glass tube:**
   Water rises due to strong adhesion with glass.
2. **Ink in a fountain pen:**
   Ink moves through narrow spaces in the nib by capillary action.
3. **Plants and trees:**
   Water moves upward from the roots to leaves through tiny xylem tubes using capillary action.
4. **Paper towel absorbing water:**
   Water travels upward through tiny pores in the paper.

5. **Oil lamp wick:**
   Oil rises through the cotton wick to feed the flame.
6. **Soil moisture:**
   Water rises in soil pores, allowing roots to absorb it even when the water table is lower.

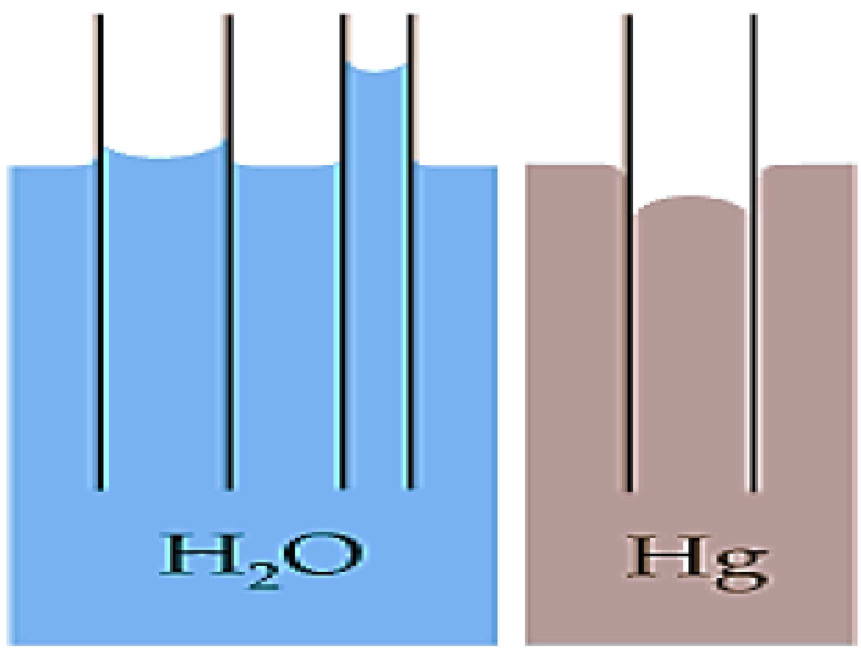
7. Example calculation

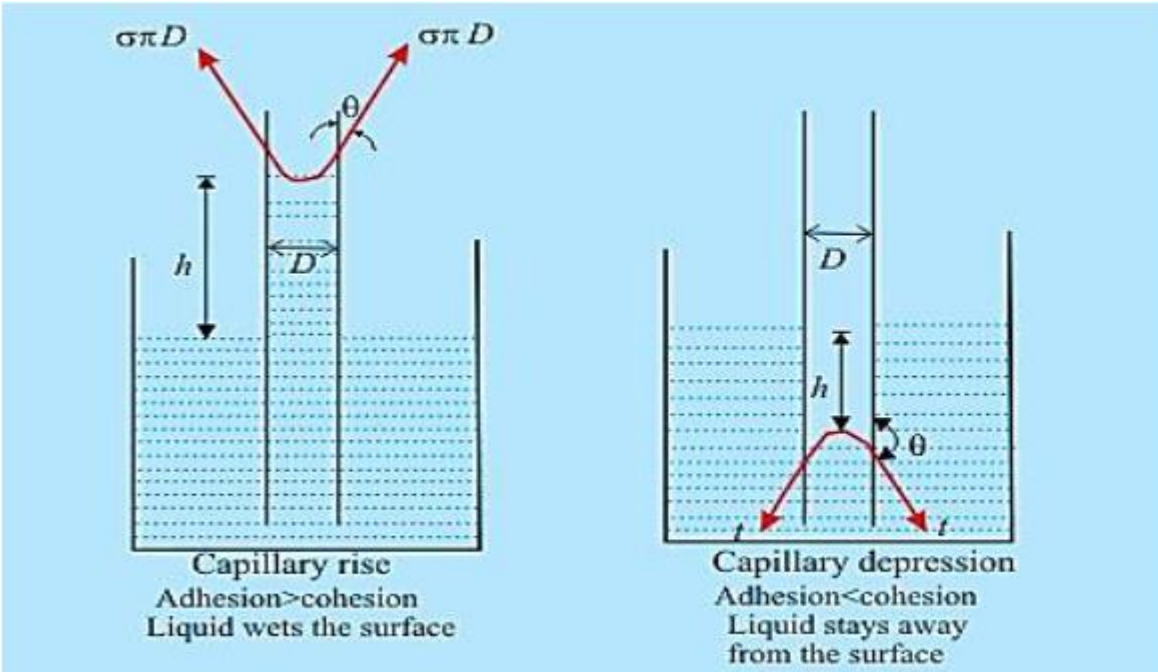
For water in a glass capillary:

Given:
$T = 0.0728$ N/m,
$\theta = 0°$,
$\rho = 1000$ kg/m³,
$r = 0.001$ m.

$$h = \frac{2(0.0728)\cos(0)}{1000(9.81)(0.001)} = 0.0148 \text{ m} = 1.48 \text{ cm}.$$

So, the water will rise about **1.5 cm** in a tube of 1 mm radius.





$$h = \frac{2T\cos\ \theta}{\rho g r}$$

This equation is commonly used to describe **capillary rise** (or depression) of a liquid in a thin tube.
Let's derive it step by step in **English**:

Step 1: Forces acting on the liquid column

When a capillary tube is placed in a liquid, the liquid either rises or falls inside the tube due to surface tension.

- The **upward force** is caused by **surface tension** Tacting along the **circumference** of the tube.
- The **downward force** is due to the **weight of the liquid column**.

At equilibrium:

$$\text{Upward Force} = \text{Downward Force}$$

Step 2: Upward force due to surface tension

Surface tension acts along the circumference of the tube.
The vertical component of this force is:

$$F_{\text{up}} = (\text{surface tension}) \times (\text{circumference}) \times \cos\ \theta$$
$$F_{\text{up}} = T \times (2\pi r) \times \cos\ \theta$$

where:

- T= surface tension (N/m)
- r= radius of the capillary (m)
- θ= contact angle between liquid and tube

Step 3: Downward force due to weight of the liquid

The weight of the liquid column of height $h$is:

$$F_{\text{down}} = \text{weight} = \text{mass} \times g$$
$$\text{mass} = \text{density} \times \text{volume} = \rho \times (\pi r^2 h)$$
$$F_{\text{down}} = \rho(\pi r^2 h)g$$

where:

- ρ= density of the liquid (kg/m^3)
- g= acceleration due to gravity (m/s^2)

Step 4: Equating forces at equilibrium

$$F_{\text{up}} = F_{\text{down}}$$
$$T(2\pi r)\cos\ \theta = \rho(\pi r^2 h)g$$

Step 5: Simplifying

Divide both sides by πr:

$$2T\cos\ \theta = \rho r h g$$

Solve for h:

$$h = \frac{2T\cos\ \theta}{\rho g r}$$

Final Derived Formula:

$$\boxed{h = \frac{2T\cos\ \theta}{\rho g r}}$$

Where:

- h= height of capillary rise (m)
- T= surface tension (N/m)
- θ= contact angle (rad or °)
- ρ= density of liquid (kg/m³)
- g= gravitational acceleration (m/s²)
- r= radius of the tube (m)

Example: Calculate the Surface tension they can get if you put mercury in the glass tube if the angle which is made with the tube equal to three degrees and intensity of mercury 13600 Kg/m3 and the tube diameter is equal to 1 centimeter and height of mercury 5 centimeter.

Solution:

$$\sigma = \frac{\rho g D h}{4\ cos\theta} = \frac{13600 * 9.81 * 0.01 * 0.05}{4 * \cos(3)} = \frac{66.708}{3.99} = 16.7\frac{N}{m}$$

Vapor Pressure of Liquids

Vapor pressure is defined as the pressure exerted by the gas-phase molecules over a liquid. Vapor pressure is a strong function of temperature—the higher the temperature, the higher the vapor pressure.

Vapor Pressure and Boiling Point:

The vapor pressure of a liquid is related to its heat of vaporization, Hvap, through **:**the Clausius-Clapeyron Equation

If P1 = 1 atm, then T1 is the normal boiling point, and we can determine the vapor pressure at any other temperature:

$$\ln {}^{P2}/_{P1} = \frac{-\Delta H_{vap}}{R}\left[\left({}^{1}/_{T_2}\right)(-{}^{1}/_{T_1})\right]$$

T1 first temp of liquid in kelvin (°C+273)

T2 Boiling temp. in kelvin

-ΔHvap change in enthalpy of vapor (kj/mol) or (kj mol $^{-1}$)

R =Gas const.=8.314 j mol K^{-1}

P1=760 mm

P2 =Pressure at boiling temp.

Example: Determine the vapor pressure of H2O at 50.0 ºC IF T2(Boiling point=100°C and Hvap = 40.79 kJ mol^{-1}

Solution:

$$\ln {}^{P2}/_{P1} = \frac{-\Delta H_{vap}}{R}\left[\left({}^{1}/_{T_2}\right)(-{}^{1}/_{T_1})\right]$$

$$\text{In } (p2 /760) = \frac{40.79}{8.314}[1/(100+273) \ - \ (50+273)]$$

P2=99.4m m.

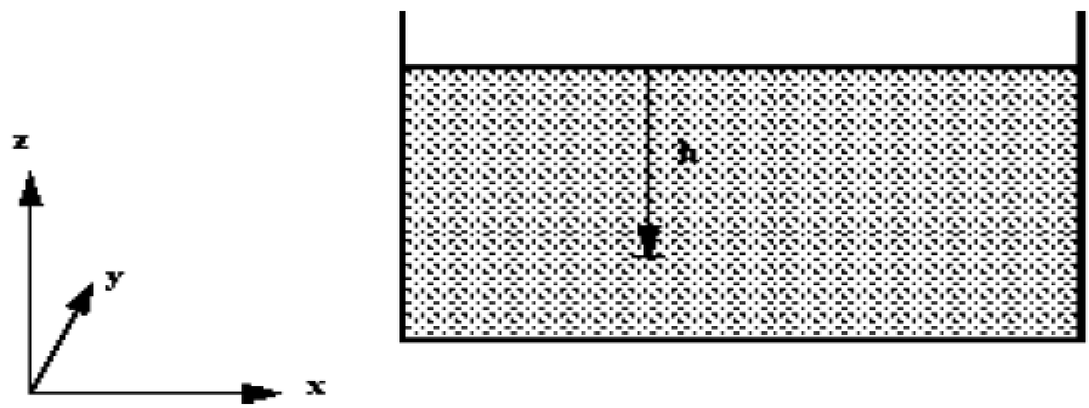
Lecture (4-6)

Pressure, liquid pressure head, Pascal law of pressure, variation of liquid pressure heat with respect to gravity, pressure at a datum for stationary liquid.

Pressure and Head

In a static fluid of constant density we have the relationship, as shown above. This can be integrated to give in a liquid with a free surface the pressure at any depth *z* measured from the free surface so that $z = -h$ (see the figure below)

Fluid head measurement in a tank.



Fluid head measurement in a tank.

This gives the pressure

$$p = \rho g h + c$$

At the surface of fluids we are normally concerned with, the pressure is the atmospheric pressure, *P* atmospheric. So

$$P = \rho g h + P_{atmospheric}$$

As we live constantly under the pressure of the atmosphere, and everything else exists under this pressure, it is convenient (and often done) to take atmospheric pressure as the datum. So, we quote pressure as above or below atmospheric. Pressure quoted in this way is known as gauge pressure i.e.

$$P_{gauge} = \rho g h$$

The lower limit of any pressure is zero - that is the pressure in a perfect vacuum. Pressure measured above this datum is known as absolute pressure i.e.

Absolute pressure = Gauge pressure + Atmospheric pressure

$$P_{absolute} = \rho g h + P_{atm}$$

As *g* is (approximately) constant, the gauge pressure can be given by stating the vertical height of any fluid of density ρ which is equal to this pressure.

$$P = \rho g h$$

This vertical height is known as **head** of fluid.

Note: If pressure is quoted in *head*, the density of the fluid *must* also be given.
Example: We can quote a pressure of 500*K N /m2* in terms of the height of a column of water of density,
ρ = 1000*kg/m3* . Using *p* = ρ*gh*

$$h = \frac{p}{\rho g} = \frac{500 * 1000}{1000 * 9.81} = 50.95\ m\ of\ water$$

And in terms of Mercury with density, $= 13600\ \frac{kg}{m^3}$

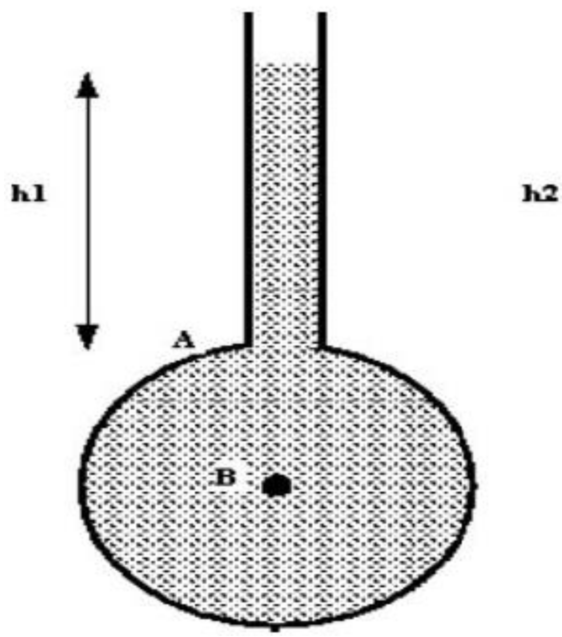
$$h = \frac{p}{\rho g} = \frac{500 * 1000}{13600 * 9.81} = 3.75\ m\ of\ mercury$$

Pressure Measurement by Manometer

The relationship between pressure and head is used to measure pressure with a manometer (also known as a liquid gauge).

The Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured. An example can be seen in the figure below. This simple device is known as a *Piezometer tube*. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is **gauge pressure**.



A simple piezometer tube manometer, Pressure at A = pressure due to column of liquid above A

$$P_A = \rho g h_1$$

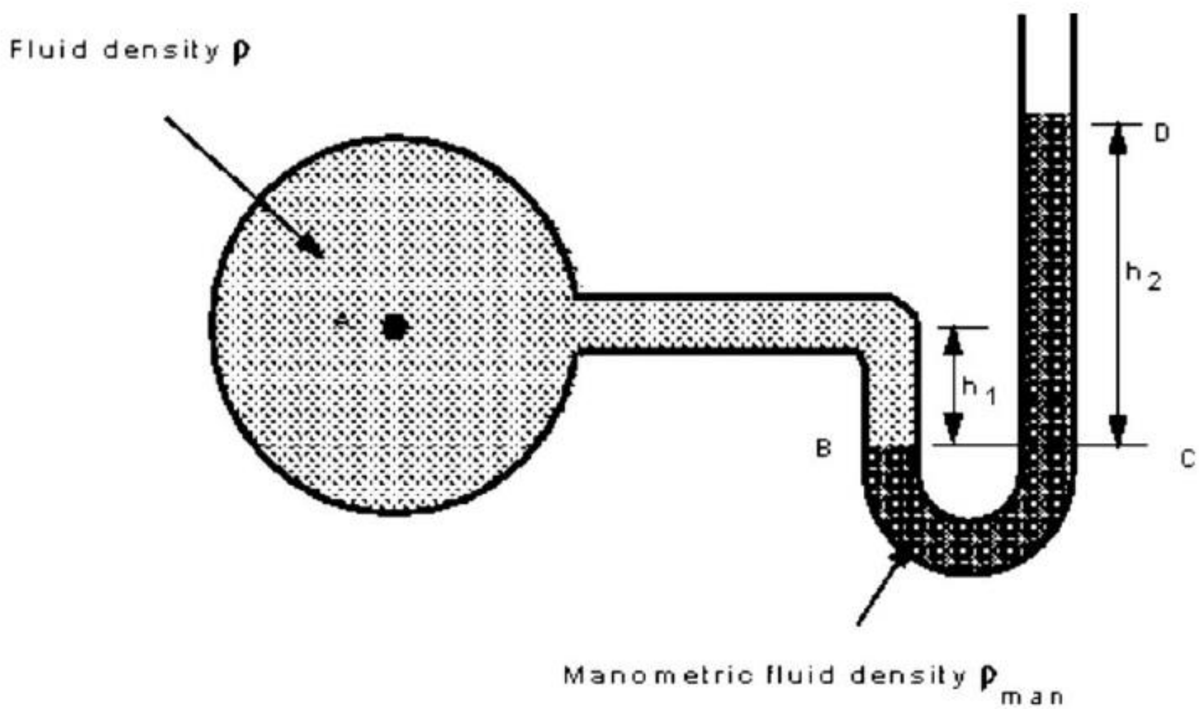
pressure at B = pressure due to column of liquid above B

$$P_B = \rho g h_2$$

This method can only be used for liquids (i.e. **not** for gases) and only when the liquid height is convenient to measure. It must not be too small or too large and pressure changes must be detectable.

The "U"-Tube Manometer

Using a ―U‖-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The ―U‖ is connected as in the figure below and filled with a fluid called the *monomeric fluid*. The fluid whose pressure is being measured should have a mass density less than that of the Monomeric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



Pressure in a continuous static fluid is the same at any horizontal level so,

Pressure at B = pressure at C

$$P_B = P_C$$

For the **left-hand** arm, pressure at B = pressure at A + pressure due to height h of fluid being measured

$$P_B = P_A + \rho g h_1$$

For the **right hand** arm , Pressure at C = pressure at D + pressure due to height h of monomeric fluid

$$P_C = P_{atm} + \rho_{man} * g h_2$$

As we are measuring *gauge pressure,* we can subtract P_{atm} giving

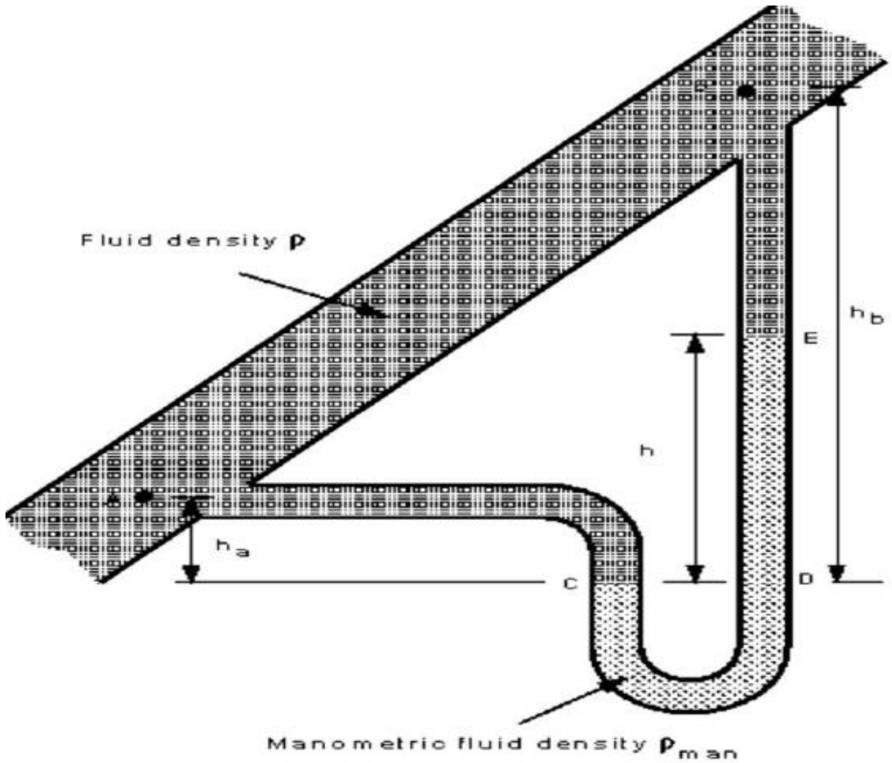
$$P_B = P_C$$

$$P_A = \rho_{man} * g * h_2 - \rho g h_1$$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the monomeric fluid i.e $\rho_{man} \gg \rho$. In this case the term$\rho g h_1$ can be neglected, and the gauge pressure given by

Measurement Of Pressure Difference Using a "U"-Tube Manometer.

If the ―U‖-tube manometer is connected to a pressurized vessel at two points the *pressure difference* between these two points can be measured.



If the manometer is arranged as in the figure above, then

$$pressure\ at\ c = pressure\ at\ D$$

$$P_C = P_D$$

$$P_C = P_A + \rho g h_a$$

$$P_D = P_B + \rho g(h_b - h) + \rho_{man} g h$$

$$P_A + \rho g h_a = P_B + \rho g(h_b - h) + \rho_{man} * g h$$

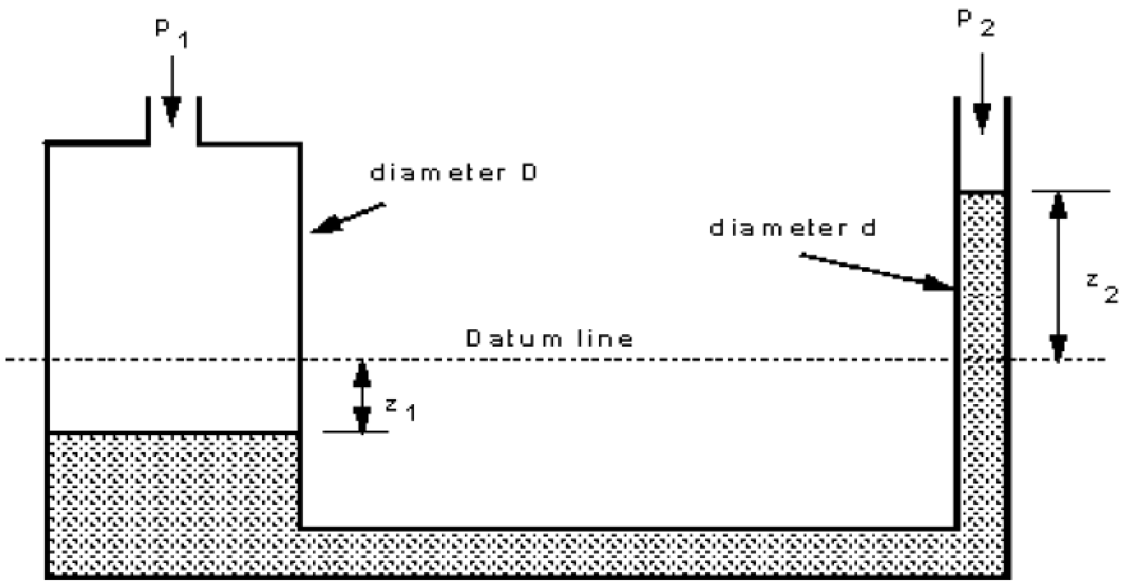
$$P_A - P_B = \rho g(h_b - h_a) + (\rho_{man} - \rho) g h$$

Again, if the fluid whose pressure difference is being measured is a gas and $\rho_{man} \gg \rho$ then the terms involving ρ can be neglected, so

$$P_A - P_B = \rho_{man} * gh$$

Advances to the "U" tube manometer.

The ―U‖-tube manometer has the disadvantage that the change in height of the liquid in both sides must be read. This can be avoided by making the diameter of one side very large compared to the other. In this case the side with the large area moves very little when the small area side move considerably more.



Assume the manometer is arranged as above to measure the pressure difference of a gas of (negligible density) and that pressure difference is P_1-P_2 If the datum line indicates the level of the monomeric fluid when the pressure difference is zero and the height differences when pressure is applied is as shown, the volume of liquid transferred from the left side to the right $= Z_2 * \left(\frac{\pi\, d2}{4}\right)$

And the fall in level of the left side is

$$z_1 = \frac{volume\ moved}{area\ of\ left\ side}$$

$$z_1 = \frac{z_2(\frac{\pi d^2}{4})}{\frac{\pi D^2}{4}}$$

$$Z_1 = Z_2(\frac{d}{D})^2$$

We know from the theory of the ―U‖ tube manometer that the height different in the two columns gives the pressure difference so

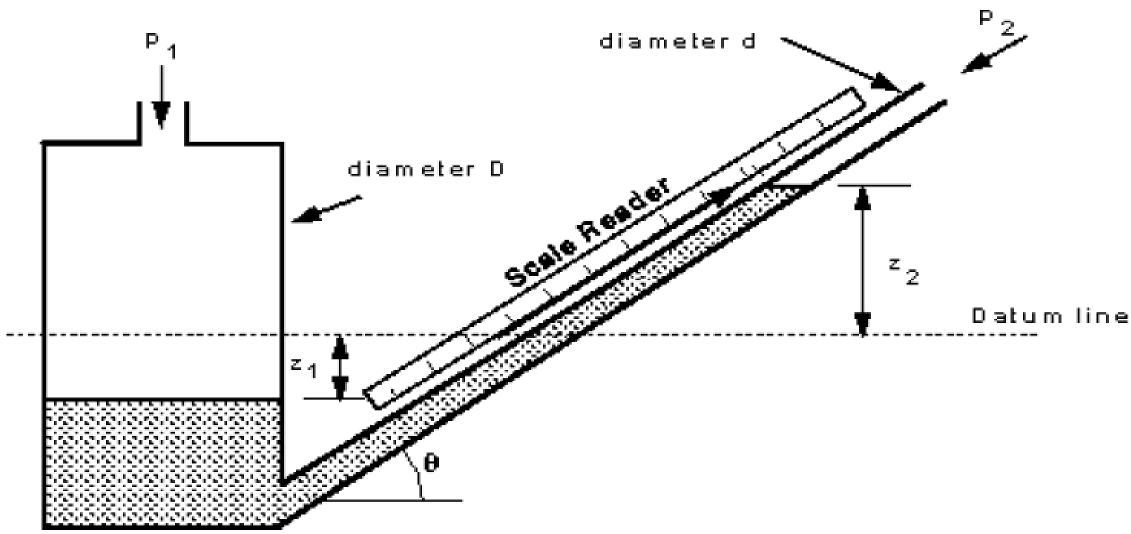
$$P_1 - P_2 = \rho g(z_2 + z_2\left(\frac{d}{D}\right)^2)$$

$$P_1 - P_2 = \rho g z_2(1 + \left(\frac{d}{D}\right)^2)$$

Clearly if D is very much larger than *d* then $(\frac{d}{D})^2$ is very small so

$$P_1 - P_2 = \rho g z_2$$

So only one reading need be taken to measure the pressure difference. If the pressure to be measured is very small then tilting the arm provides a convenient way of obtaining a larger (more easily read) movement of the manometer. The above arrangement with a tilted arm is shown in the figure below.



Tilted manometer.

The pressure difference is still given by the height change of the monomeric fluid but by placing the scale along the line of the tilted arm and taking this reading, large movements will be observed. The pressure difference is then given by

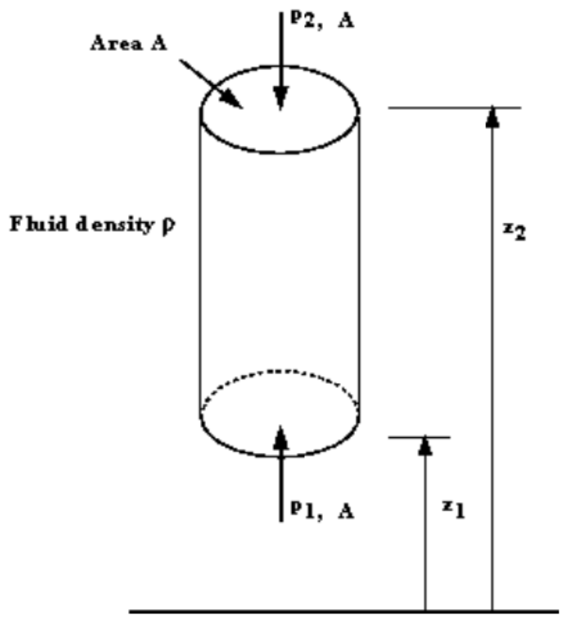
$$P_1 - P_2 = \rho g z_2$$
$$P_1 - P_2 = \rho g x\, sin\theta$$

The sensitivity to pressure change can be increased further by a greater inclination of the manometer arm, alternatively the density of the monomeric fluid may be changed.

Pascal law of pressure Pressure at any point is the same in all directions. This is known

as **Pascal's Law** and applies to fluids at rest.

Variation of Pressure Vertically in A Fluid Under Gravity



Vertical elemental cylinder of fluid

1.In a **static fluid** (a fluid at rest), the pressure **increases with depth** due to the weight of the fluid above any point.
We want to find how **pressure changes with vertical position**.

2.Consider a small fluid element

Let's take an element of fluid shaped like a vertical column (a small cylinder):

- Cross-sectional area = A
- Height = dz
- Density of fluid = ρ
- $z$is measured **upward** from some reference level.
- Pressure at the bottom face = p
- Pressure at the top face = $p + dp$(since pressure changes with height)

3. Forces on the element

- **Downward force on top face:** $(p + dp)A$
- **Upward force on bottom face:** pA
- **Weight of fluid element (downward):** $\rho gAdz$

Since the fluid is **at rest (static equilibrium)**, the sum of vertical forces = 0.

$$\text{Upward forces} = \text{Downward forces}$$

pA=(p+dp)A+ρgAdz

. Simplify the equation

Cancel $A$and rearrange:

$$0 = dp + \rho g dz$$

or

$$\boxed{\frac{dp}{dz} = -\rho g}$$

5. Interpretation

- The **negative sign** means that as z**increases upward**, p**decreases**.
- Therefore, **pressure increases downward** in a fluid under gravity.

6. Integration

For constant density ρ(incompressible fluid like water):

$$\int_{p_1}^{p_2} dp = -\rho g \int_{z_1}^{z_2} dz$$

$$\boxed{p_2 - p_1 = -\rho g(z_2 - z_1)}$$

or rearranged:

$$\boxed{p_2 = p_1 + \rho g(h)}$$

where $h = z_1 - z_2$is the **vertical depth** between the two points.

7. Final Equation (Pressure at Depth)

If the pressure at the free surface (top) is p_0,
then at depth h:

$$\boxed{p = p_0 + \rho g h}$$

8. Physical Meaning

- Pressure increases linearly with depth in a static fluid.
- It depends on **fluid density** ρ, **gravitational acceleration** g, and **depth** h.
- It does **not** depend on the shape or cross-section of the container.

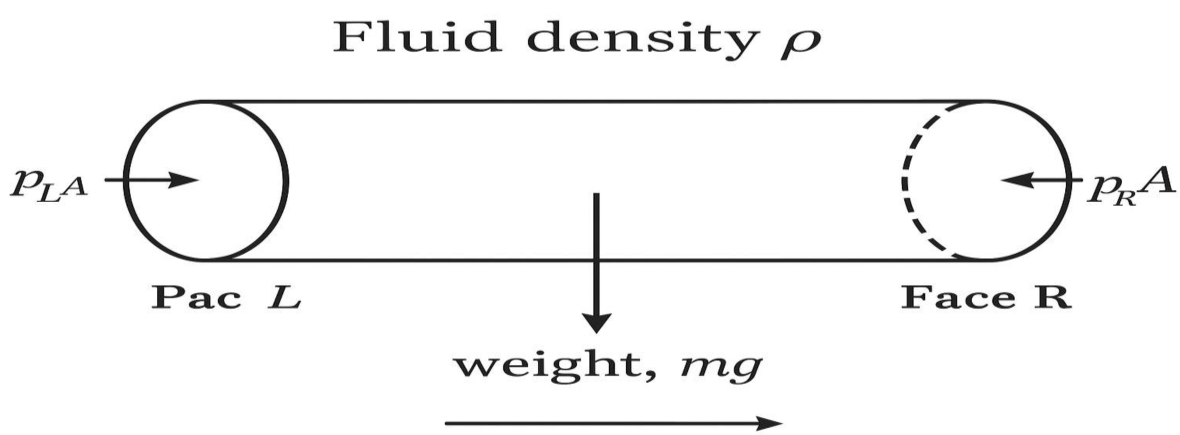
9. Example

For water ($\rho = 1000$ kg/m^3) at $h = 5$ m,
with atmospheric pressure $p_0 = 101325$ Pa:

$$p = 101325 + 1000 \times 9.81 \times 5 = 150{,}375 \text{ Pa}$$

Equality of Pressure at The Same Level in A Static Fluid

Consider the horizontal cylindrical element of fluid in the figure below, with cross-sectional area A, in a fluid of density ρ , pressure $p1$ at the left-hand end and pressure $p2$ at the right-hand end.



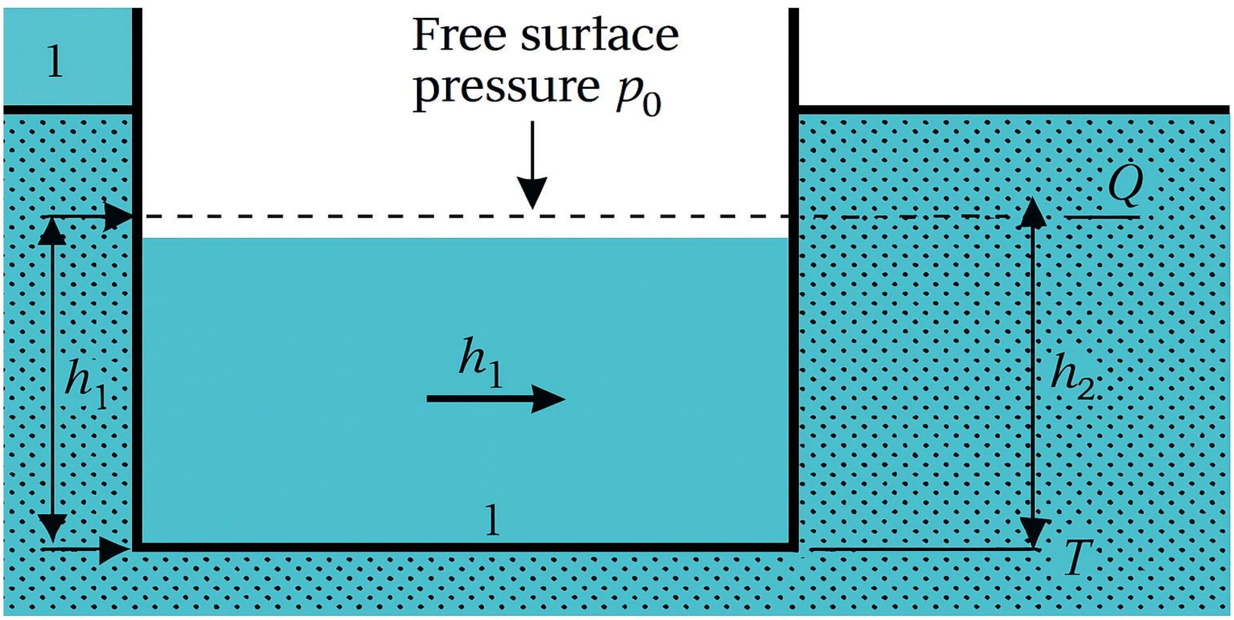
Horizontal elemental cylinder of fluid

The fluid is at equilibrium so the sum of the forces acting in the x direction is zero.

$$P_1 A = P_r A$$

$$P_1 = P_r$$

Pressure in the horizontal direction is constant. This result is the same for any *continuous* fluid. It is still true for two connected tanks which appear not to have any direct connection, for example consider the tank in the figure below Two tanks of different cross-section connected by a pipe We have shown above that and from the equation for a vertical pressure change we have This shows



Two tanks of different cross-section connected by a pipe

We have shown above that $p_l = p_r$ and from the equation for a vertical pressure change we have

$$p_l = p_p + \rho g$$

$$p_r = p_q + \rho g$$

$$p_l = p_r$$

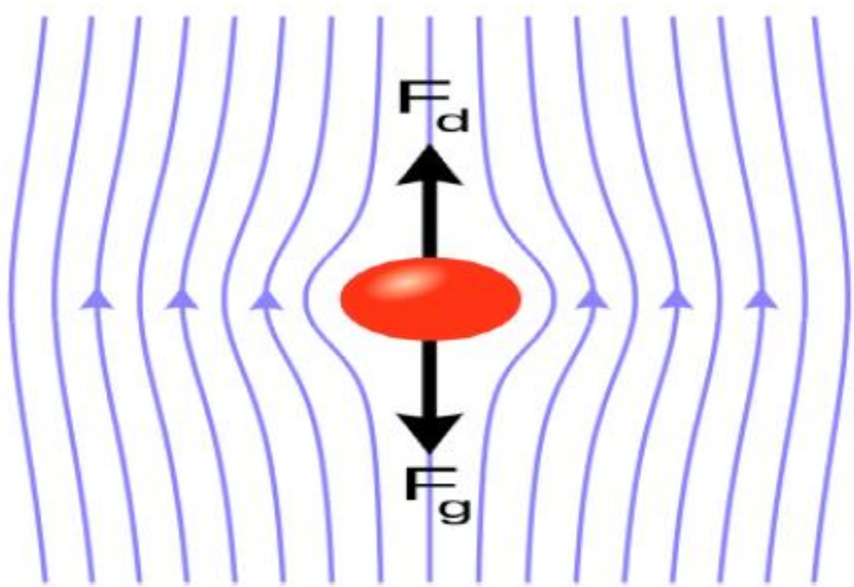
$$p_p + \rho g = p_q + \rho g$$

$$p_p = p_q$$

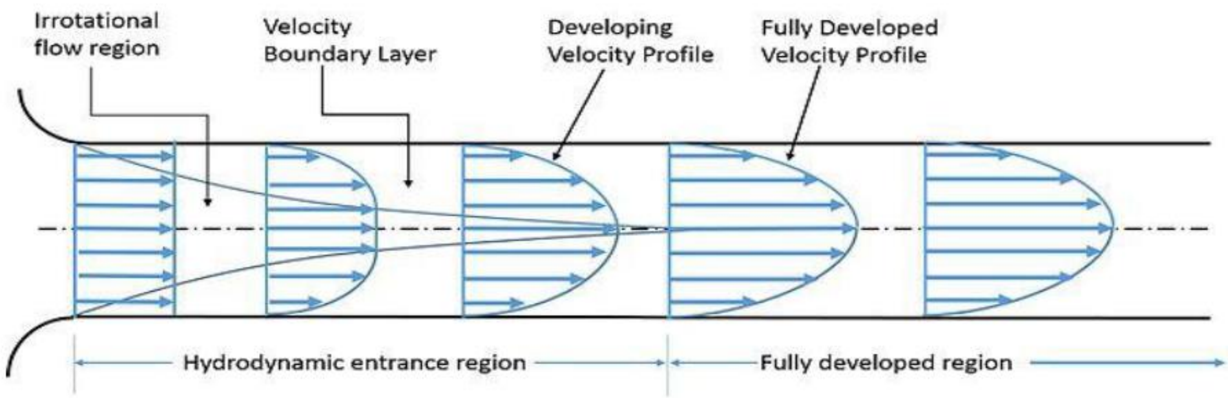
Lecture 7

Fluid motion, fluid flow, pressure of fluid flow, laminar flow, turbulent flow, velocity profile of flow, Reynold's number.

laminar flow: Occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are no cross-currents perpendicular to the direction of flow, nor eddies or swirls of fluids. In laminar flow, the motion of the particles of the fluid is very orderly with particles close to a solid surface moving in straight lines parallel to that surface. Laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection



When a fluid is flowing through a closed channel such as a pipe or between two flat plates, either of two types of flow may occur depending on the velocity and viscosity of the fluid: laminar flow or turbulent flow. Laminar flow tends to occur at lower velocities, below a threshold at which it becomes turbulent. Turbulent flow is a less orderly flow regime that is characterized by eddies or small packets of fluid particles which result in lateral mixing. In non-scientific terms, laminar flow is smooth while turbulent flow *is rough.*



Turbulence**:** or turbulent flow is a flow regime characterized by chaotic changes in pressure and flow velocity. In contrast to laminar flow, turbulence is associated with high Reynolds numbers, where inertial forces dominate viscous forces. In turbulent flow, unsteady vortices appear on many scales and interact with each other. Drag due to boundary layer friction increases. The structure and location of boundary layer separation often changes, sometimes resulting in a reduction of overall drag. This effect is exploited by aerodynamic spoilers on cars and aircraft. Turbulence is commonly observed in everyday phenomena such as surf, clouds and smoke. Most flows occurring in nature and in engineering applications are turbulent. However, turbulence has long resisted physical analysis. Richard Feynman has described it as the most important unsolved problem of classical physics.

Reynold's number *(Re)* is a dimensionless quantity that is used to help predict similar flow patterns in different fluid flow situations. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842–1912), who popularized its use in 1883. The Reynolds number is defined as the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.

Reynolds numbers frequently arise when performing scaling of fluid dynamics problems, and as such can be used to determine dynamic similitude between two different cases of fluid flow. They are also used to characterize different flow regimes within a similar fluid, such as laminar or turbulent flow:

• laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion;

• Turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

$$R_e = \frac{\rho * v * d}{\mu} \quad , \quad R_e = \frac{\rho * v * l}{\mu}$$

Example:10

When water is running in a round tube of diameter 3 cm at a flow velocity of 2m/s, is this flow laminar or turbulent? Assume that the kinematic viscosity of water is 1 x10-6 m^2/s.

solution:

$$\boldsymbol{Re = \frac{\rho v d}{\mu} = \frac{v d}{V}}$$

(V is kinematic viscosity)

Re= 2 x 0.03 / 1 x 10^{-6}

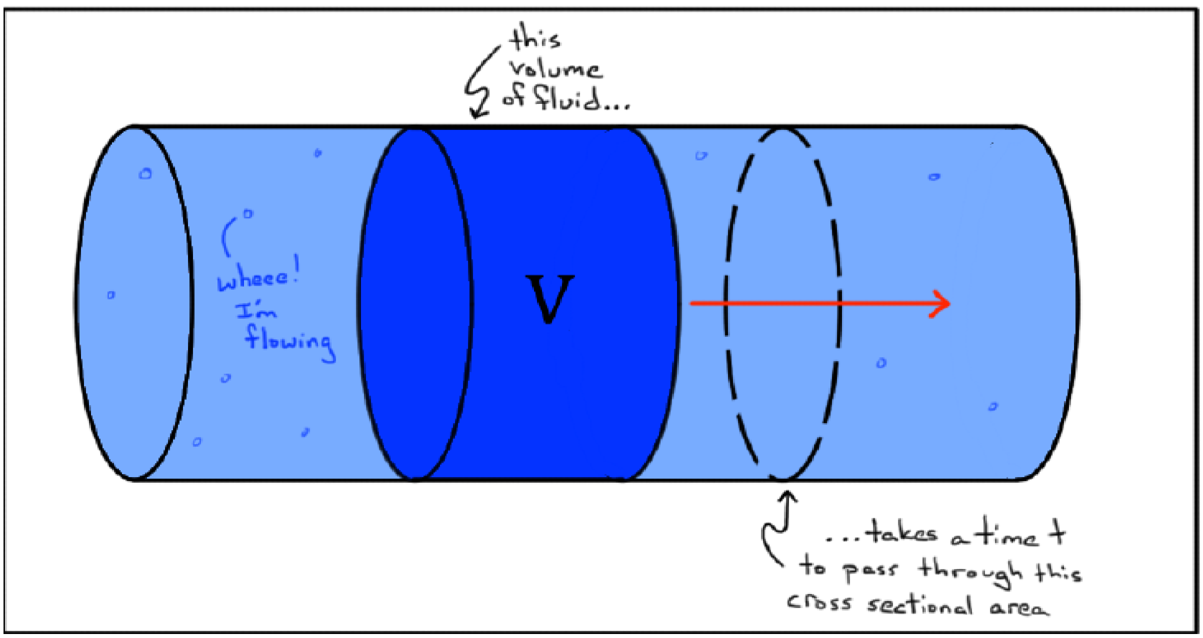
=60000>2320

Then the flow is turbulent

Lecture 8

Flow rate, volumetric flow rate, mass flow rate, - Continuity equation, problems on continuity equation for uncompressible fluids. ***What does volume flow rate mean?***

You might hear the term **volume flow rate** and think it sounds boring, but volume flow rate keeps you alive. I'll tell you how in a second, but first we should define volume flow rate. The volume flow rate Q of a fluid is defined to be the volume of fluid that is passing through a given cross sectional area per unit time. The term cross sectional area is just a fancy term often used to describe the area through which something is flowing, e.g., the circular area inside the dashed line in the diagram below



Since volume flow rate measures the amount of volume that passes through an area per time, the equation for the volume flow rate looks like this:

$$Q = \frac{\text{volume}}{\text{time}} = \frac{V}{t}$$

In S.I. units (International System of Units), volume flow rate has units of meters cubed per second, since it tells you the number of cubic meters of fluid that flow per second.

Is there another formula for volume flow rate*? It turns out there's a useful alternative to writing the volume flow rate as*

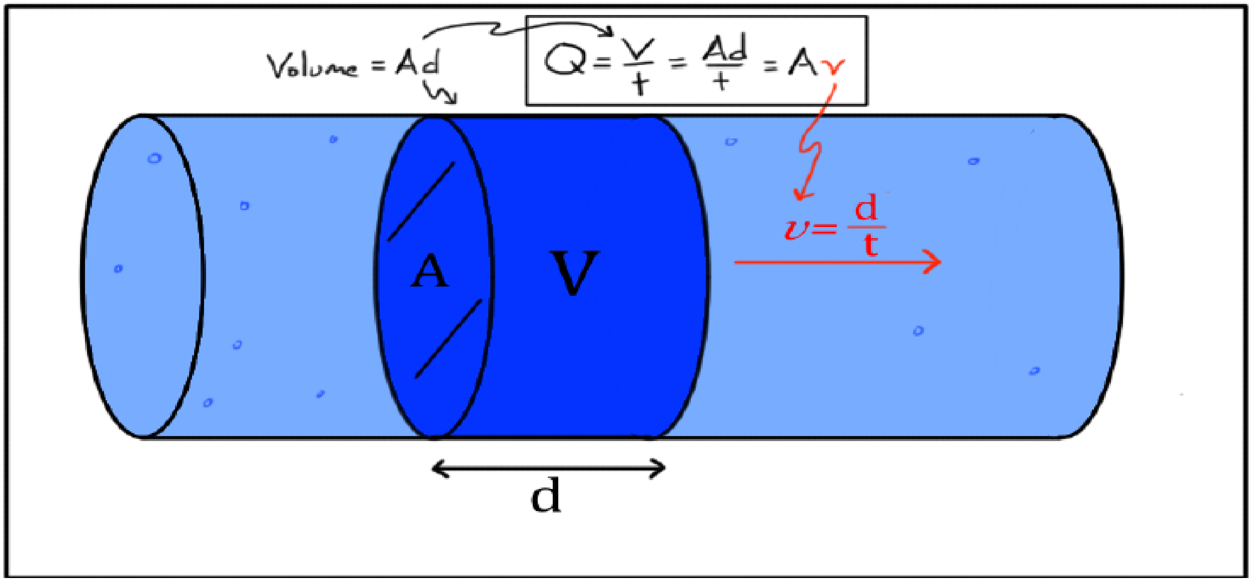
$$Q = \frac{volume}{time} = \frac{v}{t} = \frac{A * d}{t} = \frac{A}{t} * d$$

The volume of a portion of the fluid in a pipe can be written as $v = A * d$ *, where A is the cross sectional area of the fluid and d is the width of that portion of fluid, see the diagram below. We can substitute this formula for volume* ***v*** *into the volume flow rate to get the following*

But the term $\frac{d}{t}$ *is just the length of the volume of fluid divided by the time it took the fluid to flow through its length, which is just the speed of the fluid. So we can replace* $\frac{d}{t}$ *d, divided by, t, the previous equation and get*

$$Q = \frac{volume}{time} = \frac{v}{t} = \frac{A * d}{t} = \frac{d}{t} * A = V * A$$

$$Q = V * A$$



Mass flow rate ($\dot{m}$)

$$Q = VA = \frac{m}{sec} * m^2 = \frac{m^3}{sec}$$

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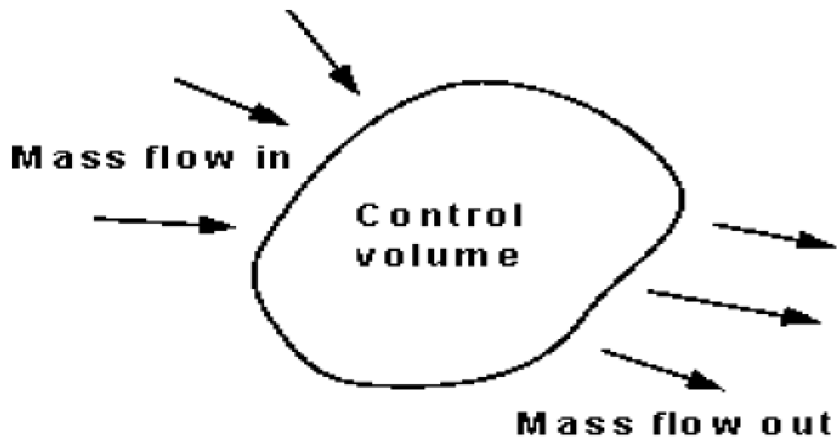
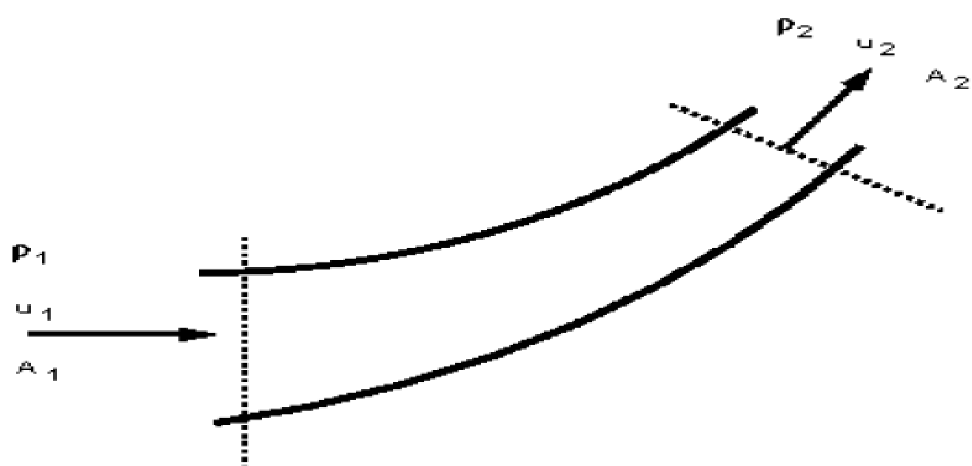
$$\rho Q = \rho VA = \frac{kg}{m^3} * \frac{m}{sec} * m^2 = \frac{kg}{sec}$$

Continuity

Matter cannot be created or destroyed - (it is simply changed in to a different form of matter). This principle is known as the *conservation of mass* and we use it in the analysis of flowing fluids. The principle is applied to fixed volumes, known as control volumes (or surfaces), like that in the figure below:



For any control volume the principle of *conservation of mass* says Mass entering per unit time = Mass leaving per unit time + Increase of mass in the Control volume per unit time for **steady** flow there is no increase in the mass within the control volume, so Mass entering per unit time = Mass leaving per unit time This can be applied to a stream tube such as that shown below. No fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of
this stream tube section.



mass entering per unit time at end 1 = mass leaving per unit time at end 2 The flow of fluid through a real pipe (or any other vessel) will vary due to the presence of a wall – in this case we can use the *mean* velocity and write

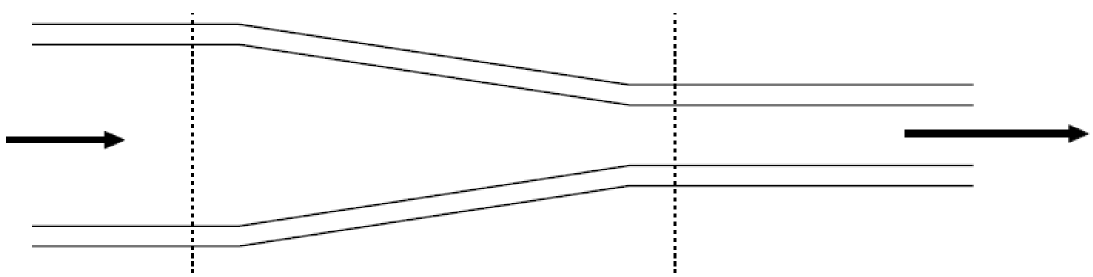
$$\rho_1 * u_1 * A_1 = \rho_2 * u_2 * A_2 = \dot{m}$$

$$Q = u_1 * A_1 = u_2 * A_2$$

This is the form of the continuity equation most often used.

Some example applications

We can apply the principle of continuity to pipes with cross sections which change along their length. Consider the diagram below of a pipe with a contraction:



A liquid is flowing from left to right and the pipe is narrowing in the same direction. By the continuity principle, the *mass flow rate* must be the same at each section - the mass going into the pipe is equal to the mass going out of the pipe. So, we can write:

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 * u_1 * A_1 = \rho_2 * u_2 * A_2 = \dot{m}$$

(with the sub-scripts 1 and 2 indicating the values at the two sections) As we are considering a liquid, usually water, which is *not* very compressible, the density changes very little so we can say $\rho = \rho_1 = \rho_2$ this also says that the *volume flow rate* is constant or that

$$Q_1 = Q_2$$
$$u_1 * A_1 = u_2 * A_2$$

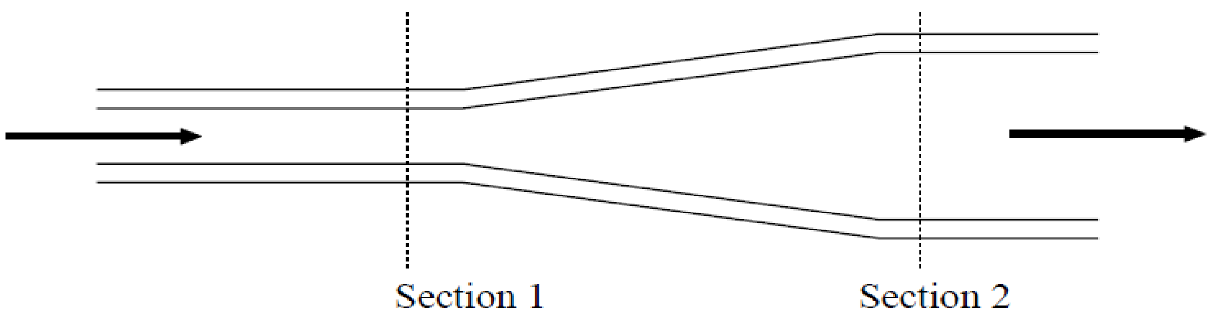
Example: if the area $A_1 = 10 * 10^{-3}\ m^2 and\ A_2 = 3 * 10^{-3} m^2$ and the upstream mean velocity,
$u_1 = 2.1 \frac{m}{sec}$, then the downstream mean velocity can be calculated by

$$u_2 = \frac{u_1 * A_1}{A_2} = \frac{2.1 * 10 * 10^{-3}}{3 * 10^{-3}} = 7\ \frac{m}{sec}$$

Notice how the downstream velocity only changes from the upstream by the ratio of the two areas of the pipe. As the area of the circular pipe is a function of the diameter we can reduce the calculation further,

$$u_2 = \frac{A_1}{A_2} u_1 = \frac{\frac{\pi d_1^2}{4}}{\frac{\pi d_2^2}{4}} u_1 = \frac{d_1^2}{d_2^2} u_1 = (\frac{d_1}{d_2})^2 u_1$$

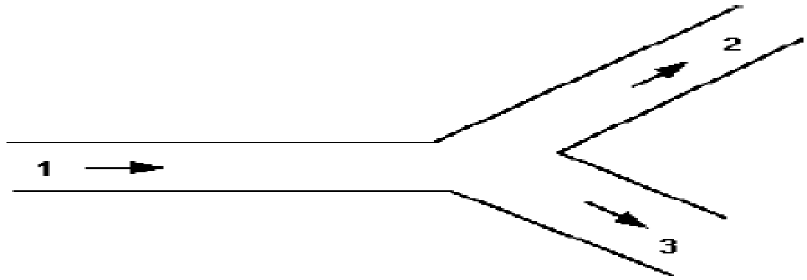
Now try this on a *diffuser,* a pipe which expands or diverges as in the figure below,



If the diameter at section 1 is $d_1 = 30mm$ and at section 2 $d_2 = 40mm$
is $u_2 = 3.0 m/s$. The velocity entering the diffuser is given by, and the mean velocity at section 2

$$u_1 = (\frac{40}{30})^2 * 3 = 5.3\ \frac{m}{sec}$$

Another example of the use of the continuity principle is to determine the velocities in pipes coming from a junction.



$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

$$\rho_1 * u_1 * A_1 = \rho_2 * u_2 * A_2 + \rho_3 * u_3 * A_3$$

$$\rho_1 = \rho_2 = \rho_3$$

$$u_1 * A_1 = u_2 * A_2 + u_3 * A_3$$

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter 60mm. What are the values of discharge and mean velocity in each pipe?

$$Q_1 = u_1 * A_1 = u_1 * \frac{\pi d_1^2}{4} = 0.00392 \frac{m^3}{sec}$$

$$Q_2 = 0.3Q_1 = 0.3 * 0.00392 = 0.0011 \frac{m^3}{sec}$$

$$Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - 0.3Q_1 = 0.00392 - 0.3 * 0.00392 = 0.00275 \frac{m^3}{sec}$$

Lecture (9-11)

Bernoulli equation and application.

The Bernoulli Equation - Work and Energy

Work and energy

We know that if we drop a ball, it accelerates downward with an acceleration $g = 9.81 m/s^2$ (neglecting the frictional resistance due to air). We can calculate the speed of the ball after falling a distance h by the formula $v^2 = u^2 + 2as$ ($a = g$ and $s = h$). The equation could be applied to a falling droplet of water as the same laws of motion apply A more general approach to obtaining the parameters of motion (of both solids and fluids) is to apply the principle of ***conservation of energy***. When friction is negligible the Sum of kinetic energy and gravitational potential energy is constant.

$$kinatic\ energy = K.E = \frac{1}{2} * m * u^2$$

$$\text{Gravitational potential energy} = \mathrm{m * g * h}$$

(m is the mass, v is the velocity and h is the height above the datum).

To apply this to a falling droplet we have an initial velocity of zero, and it falls through a height of initial kinetic energy = 0 ,

Initial potential energy = $mg\,h$

$$final\ kinatic\ energy = K.E = \frac{1}{2} * m * u^2$$

Final potential energy = 0

We know that

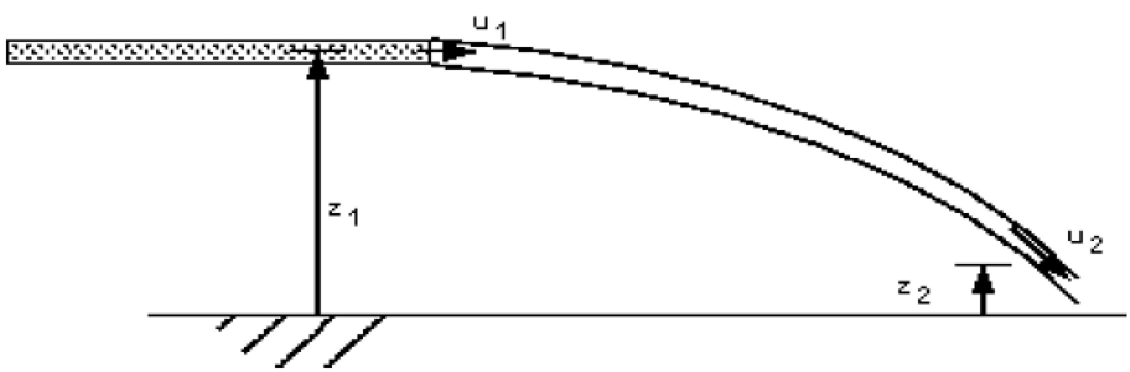
kinetic energy + potential energy = constant

Initial kinetic energy + Initial potential energy = Final kinetic energy + Final potential energy

$$m * g * h = \frac{1}{2} * m * u^2$$

$$u = \sqrt{2gh}$$

Although this is applied to a drop of liquid, a similar method can be applied to a **continuous jet** of liquid.



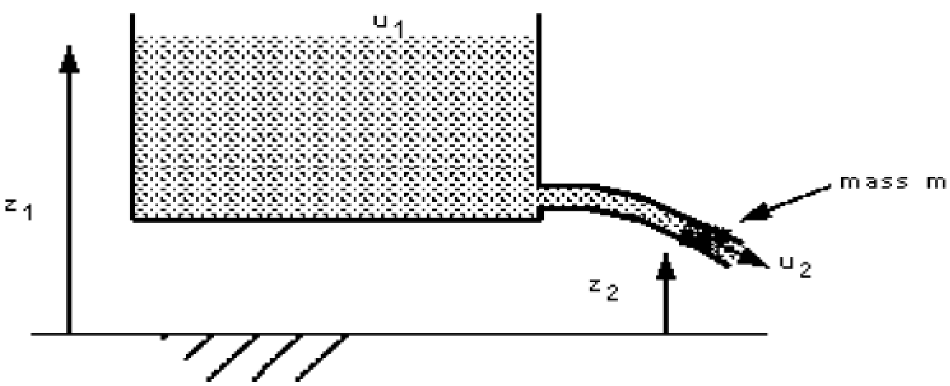
We can consider the situation as in the figure above - a continuous jet of water coming from a pipe with velocity $u1$. One particle of the liquid with mass m travels with the jet and falls from height $z1$ to $z2$. The velocity also changes from $u1$ to $u2$. The jet is travelling in air where the pressure is everywhere atmospheric so there is no force due to pressure acting on the fluid. The only force which is acting is that due to gravity. The sum of the kinetic and potential energies remains constant (as we neglect energy losses due to friction) so

$$m * g * z_1 + \frac{1}{2} * m * u_1^2 = m * g * z_2 + \frac{1}{2} * m * u_2^2$$

$$g * z_1 + \frac{1}{2} * u_1^2 = g * z_2 + \frac{1}{2} * u_2^2$$

Flow from a reservoir

We can use a very similar application of the energy conservation concept to determine the velocity of flow along a pipe from a reservoir. Consider the _idealized reservoir 'in the figure below.



The level of the water in the reservoir is $z1$. Considering the energy situation -there is no movement of water so kinetic energy is zero but the gravitational potential energy is $mgz1$.

If a pipe is attached at the bottom water flows along this pipe out of the tank to a level $z2$. A mass m has flowed from the top of the reservoir to the nozzle and it has gained a velocity $u2$

The kinetic energy is $\frac{1}{2} m u_2^2$and the potential energy $m.g.z_2$Summarizing

Initial kinetic energy = 0

Initial potential energy= $mgz1$

Final kinetic energy $\frac{1}{2}mu_2^2$

Final potential energy = $m.g.z_2$

We know that

kinetic energy + potential energy = constant

$$mgz_1 = \frac{1}{2}mu_2^2 + mgz_1 \rightarrow mg(z_1 - z_2) = \frac{1}{2}mu_2^2$$

$$u_2 = \sqrt{2g(z_1 - z_2)}$$

We now have a expression for the velocity of the water as it flows from of a pipe nozzle at a height ($z1 - z2$) below the surface of the reservoir. (Neglecting friction losses in the pipe and the nozzle).

Now apply this to this example: A reservoir of water has the surface at 310m above the outlet nozzle of a pipe with diameter 15mm. What is the a) velocity, b) the discharge out of the nozzle and c) mass flow rate. (Neglect all friction in the nozzle and the pipe).

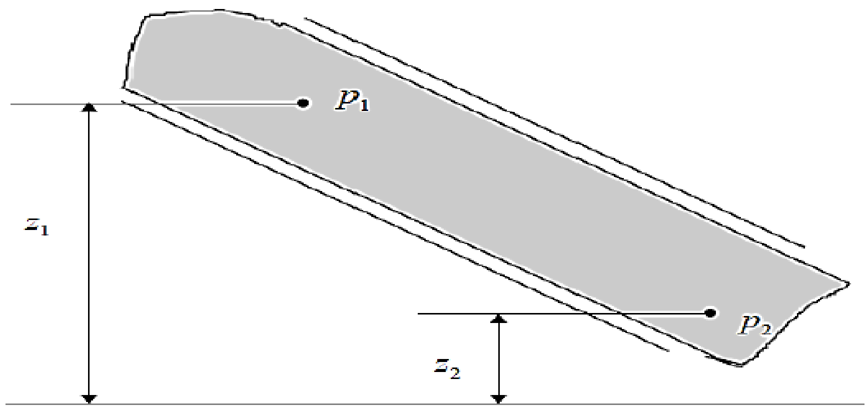
$$u_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2 * 9.81 * 310} = 78 \frac{m}{sec}$$

Volume flow rate is equal to the area of the nozzle multiplied by the velocity

$$Q = uA = \frac{\pi * 0.015^2}{4} * 78 = 0.01378 \frac{m^3}{sec}$$

The density of water is 1000 $kg / m3$ so the mass flow rate is

In the above examples the resultant pressure force was always zero as the pressure surrounding the fluid was the everywhere the same - atmospheric. If the pressures had been different there would have been an extra force acting and we would have to take into account the work done by this force when calculating the final velocity. We have already seen in the hydrostatics section an example of pressure difference where the velocities are zero.



The pipe is filled with stationary fluid of density ρ has pressures $p1$ and $p2$ at levels $z1$ and $z2$ respectively. What is the pressure difference in terms of these levels?

$$P_2 - P_1 = \rho g(z_1 - z_2)$$

$$\frac{P_1}{\rho} + gz_1 = \frac{P_2}{\rho} + gz_2$$

This applies when the pressure varies but the fluid is stationary. Compare this to the equation derived for a moving fluid but constant pressure:

$$\frac{1}{2}u_1^2 + gz_1 = \frac{1}{2}u_2^2 + gz_2$$

You can see that these are similar form. What would happen if both pressure and velocity varies.

4.3.2 Energy Equation (Burnoulli's Equation).

Assumptions:-

1. Derived depending on Conservation of energy law.
2. Steady flow.
3. Uniform flow.
4. Ideal flow (frictionless).
5. Incompressible flow.

Burnoulli's Equation states as follow :-

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line and the total energy heads must be constant".

By considering the given figure:-

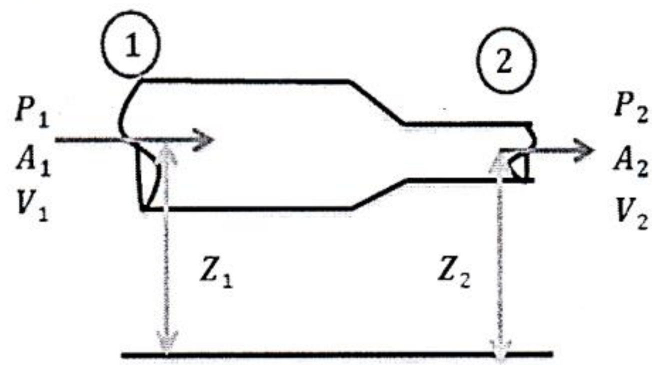
$\frac{P}{\gamma}+\frac{V^2}{2g}+Z=constant=H$ **Where:-**

H:- **total head, (m)**

$\frac{P}{\gamma}$:- **pressure head, (m)**

$\frac{V^2}{2g}$:- **velocity head, (m)**

Z :- **potential head, (m)**

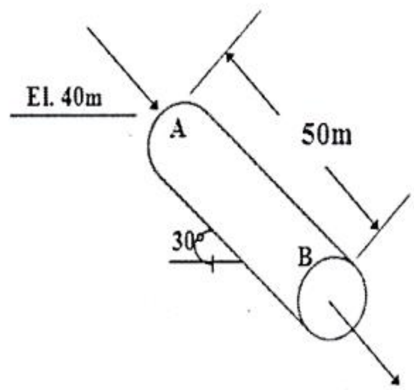


Then :-

$$\frac{P_1}{\gamma}+\frac{V^2{}_1}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V^2{}_2}{2g}+Z_2$$ ….. **Burnoulli's Equation**

Example(4.1): Water flows through a cylindrical pipe, 150mmd. If the pressure at point A is 130Kpa, what is the pressure at B.

Solution:-



$$Z_A = 40m$$

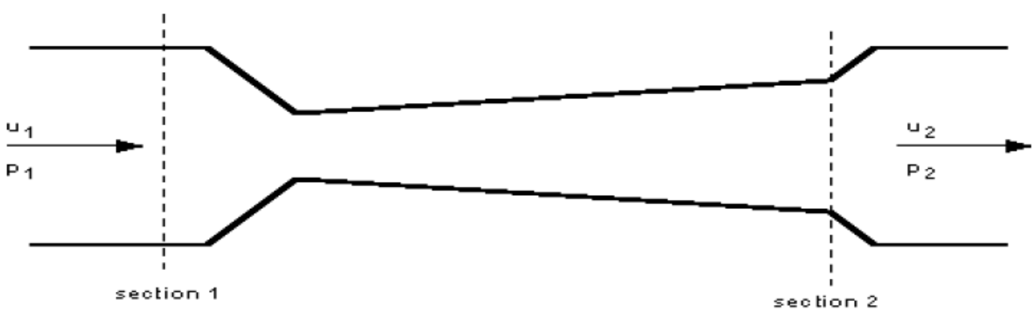
$$Z_B = 40 - 50\sin e30 = 40 - 25 = 15m$$

$$\frac{V_A^{\ 2}}{2g} + \frac{P_A}{\gamma} + Z_A = \frac{V_B^{\ 2}}{2g} + \frac{P_B}{\gamma} + Z_B$$

$$\frac{130*10^3}{9800} + 40 = \frac{P_B}{9800} + 15 \qquad (V_A = V_B)$$

$$P_B = 375Kpa \qquad Ans.$$

A fluid of constant density ρ = 960 $kg / m3$ is flowing steadily through the above tube. The diameters at the sections are $d1$ =100 mm and $d2$ = 80mm. The gauge pressure at 1 is $p1 = 200kN / m$ And velocity here is $u1 = 5m / s$. We want to know the gauge pressure at section 2.



We shall of course use the Bernoulli equation to do this and we apply it along a streamline joining section 1 with section 2. The tube is horizontal, with $z1 = z2$ so Bernoulli gives us the following equation for pressure at section 2:

$$p_2 = p_1 + \frac{\rho}{2}(u_1^2 - u_2^2)$$

But we do not know the value of *u*2. We can calculate this from the continuity equation: Discharge into the tube is equal to the discharge out i.e.

$$A_1 u_1 = A_2 u_2 \quad \rightarrow \quad u_2 = \frac{A_1}{A_2} * u_1 = \left(\frac{d_1}{d_2}\right)^2 * u_1 = \left(\frac{100}{80}\right)^2 * 5 = 7.81 \frac{m}{sec}$$